EXPERIMENTS

ON THE

FLEXURE OF BEAMS,

RESULTING IN THE

DISCOVERY OF NEW LAWS OF FAILURE BY BUCKLING.

BY

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PREFACE.

WE begin in another column the publication of a short series of articles on "The Flexure of Beams," which, for the first time in many years, breaks new ground. The analysis and presentation of this subject in treatises and text-books are almost stereotyped. No important contribution to the stock of knowledge of the subject has been made for many years, the chapter being apparently closed, and this in spite of the fact that certain methods of action and of failure of beams are well known, though not recognized in any existing formula. The study of the failure of beams by the buckling of the compression side has been strangely neglected, and now that it has been taken up, it proves to be the central fact and key to the entire subject when looked at in the broadest sense.

The analogy of the failure of the compression side of a beam by buckling to the method of failure of a long column was, of course, long ago remarked, but we believe there has been no previous attempt—certainly no successful attempt—to connect the two by a formula.

Mr. Guy's experiments have been successful beyond any reasonable expectation in connecting them and in showing that Euler's formula for long columns is, in fact, the fundamental formula which lies at the base of the whole subject.

We believe there is not in any existing text-book a formula by which the strength of a long beam unsupported sideways may be determined. Mr. Guy discloses the laws of failure in this manner for the first time, and they are in consequence entitled to be called, as we shall hereafter call them, Guy's laws. The disclosure of such laws alone would be a notable achievement, but when to this is added the connection through definite formulas of long beams and columns, including inclined beams acted upon by vertical forces, the accomplishment is nothing less than brilliant.

Mr. Guy's articles are a distinct and notable addition to the science of engineering, and we consider that to be the means of their publication is a rare privilege.

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EXPERIMENTS ON THE FLEXURE OF BEAMS.

INTRODUCTION.

THE aim of the engineer is to build strongly and economically — in other words, to obtain a maximum effect with a given amount of material.

When the design of a machine is outlined, the various mechanisms being shown in place, the most important thing to do is to determine accurately the positions, directions and intensities of the forces applied to and developed by the machine. This done, the necessary sizes of the different parts are calculated according to the rules established by the science of the Resistance of Materials, and finally, feeling confident that his work will be strong enough within the limits just found, the designer judiciously modifies the shapes of the parts, in order to please his taste and to insure a neat appearance to the finished machine.

But when he began to materialize his conception he met a great difficulty: those rules apply only to well-

defined cases; and even then, notwithstanding scientific researches and numberless experiments, many of the formulas deduced therefrom and offered in our text-books are difficult to apply, because of their indeterminate form. Often the designer has recourse to his experience, which undoubtedly serves him well when the case at hand has some similarity with those he has treated before, but which, at times, leaves him helpless before some unraveled combination of molecular forces. To loosen this new Gordian knot two instruments can be used—one, the mathematical analysis, and the other carefully conducted experiments. The first of these is beyond the reach of the majority of engineers, but in order to be producive of successful results it must be corroborated by the second. To conduct experiments is a most delicate task, which before being undertaken requires much thought. Just as the analysis has a basis which is the study of the deformations of a molecule of matter caused by the various forces at play thereon, in the same way the objective basis of the experiments is the underlying principle, simple or complex, governing the case. The saying that every rule has its exceptions applies here; as with grammar, we must study first the one and then the others. The principle, once discovered, becomes a pole whence radiate an infinity of combinations. It is clear that, knowing their origin, the latter can readily be investigated. In practice the combina-

tions are most generally met with, and the principle very rarely. This explains why many experimenters, mistaking the exception for the rule, have vainly tried to establish formulas generalizing the results of their investigations.

Therefore, since the science of the Resistance of Materials is primarily one of observation, it can be greatly furthered by the individual efforts of all those who have the rare opportunity to make careful and extensive tests. With this object in view, after many unsuccessful attempts to solve analytically a familiar question in flexure, yet convinced of the possibility of an exact solution and desirous of acquiring reliable experience, the writer undertook several series of experiments, the first of which is exposed in the following. He was thus fortunate in discovering some laws which had escaped investigation and which will be found of the greatest utility.

Posing as a problem the simplest case of flexure of a solid, a case of everyday occurrence to the designer, it will be shown, first, that the ordinary formulas are insufficient to solve it; then, the new laws being explained and applied, a complete solution will be obtained.

A SIMPLE PROBLEM.

It is required to design a beam having a minimum volume and which, firmly held at one end, must sustain at the other a load P acting perpendicularly to the length L.

To solve this problem it is necessary to know:
1—The kind of material composing the beam;
2—The modulus of elasticity of the material;
3—Its limits of elasticity;
4—The factors of safety, or ratios between the limits of elasticity and the longitudinal tension or compression which the fibers most remote from the neutral axis will be allowed to bear.

Any section distant a length x from the loaded end of the beam must resist a bending moment $P x$ and a shearing force P; but as the latter is not very considerable, compared with the former, it is customary, in practice, to neglect it in the calculations, and provide for it only at the point of application of the load.

Since the beam must sustain the load, the ultimate strength of the material is not a factor in the question, and the allowable stress per unit of area must be well within the limits of elasticity.

In accordance with the common theory of flexure the following assumptions are made:

1—The allowable tensile and compressive stresses of the material are equal and termed f;

2—The stresses in any section of the beam are directly proportional to their distance from the neutral axis.

The equation expressing the relation between the exterior forces acting on the beam and the molecular forces developed by the former in a given section of the beam is:

$$P x = f \frac{I}{a} \tag{1}$$

$\frac{I}{a}$ is termed the moment of resistance of a section having a moment of inertia I; and a is the distance from the neutral axis to the most strained fiber in the section.

When f is made constant in all the sections the beam is a solid of uniform resistance, and if each section has a minimum area the beam has a minimum volume.

Of the plain sections, the most employed are the circular, the square, and the rectangular. If the first or second is chosen the problem is partially solved, because the expression of the moment of resistance contains then only one unknown quantity, which can easily be found by assigning to x a numerical value. But the rectangular form is very much more economical than the other two, since, in varying one of its dimensions, the area can

become very small and at the limit be zero. Effectively, for a rectangle the moments of inertia and of resistance are respectively:

$$I = \frac{b\,h^3}{12} \qquad \frac{I}{a} = \frac{b\,h^2}{6}$$

in which expressions b is the base and h, parallel to the direction of the load, is the hight of the section.

Replacing in equation (1) $\frac{I}{a}$ by its value as above we have:

$$P\,x = f\frac{b\,h^2}{6}.$$

For a chosen value of x,

$$b\,h^2 = \frac{6\,P\,x}{f}, \tag{2}$$

and it remains to find the ratio of b to h which will make the area $A = b\,h$ a minimum.

The expression (2) is indeterminate since the factors b and h are both unknown. In practice a value is assumed for one, and the other is thereby determined; but knowing that the greater the side h the smaller is the area, the designer is much embarrassed, for, if the ratio $\frac{h}{b}$ is small, the area of the section is considerable, and if the ratio is large the beam may be so thin that it will warp laterally and be thus unable to sustain the load.

At this point a search for help through the text-books results in bringing out for consideration:

(1) The transverse shearing, which we have neglected at first;

(2) The longitudinal shearing;

(3) The deflection of the beam.

THE TRANSVERSE SHEARING.

In this case any section must be large enough to resist the shearing force P; hence, if f_s is the allowable shearing stress, we must have:

$$\frac{P}{f_s} = b\,h \qquad (3)$$

This equation insures strength enough at the loaded end of the beam, but evidently does not provide against lateral warping. Should this area (3) be adopted equations (2) and (3) combined give:

$$b\,h = \frac{6\,P\,x}{f\,h} = \frac{P}{f_s}$$

and $$h = \frac{6 f_s}{f}\,x$$

Generally,

$$f_s = \frac{4}{5}\,f, \text{ whence } h = \frac{6 \times 4}{5}\,x = 4.8\,x.$$

We have thus the ridiculous result that the hight of the section at the danger point must be nearly five times greater than the leverage of the bending force. It is evident that equation (3) cannot help in solving the problem. However, it must be admitted that the method followed here, although employed by some authorities, is not very correct, for, really, the expressions (2) and (3) instead of being equated should be combined.

THE LONGITUDINAL SHEARING.

The longitudinal shearing force is that which tends to destroy the adhesion of the laminæ of which the beam is assumed to be formed.

Mr. Jouravski was the first who took into consideration a longitudinal shearing force, maximum in the neutral plane of the beam. (See "Annales des Ponts et Chaussées, 1856).

Professor Reuleaux treats this subject in his "Constructor" as follows: "Since in a deflected beam there is on the tension side a continual tension, on the compression side a continual compression of the fibers, it follows that the neutral plane is subjected to a shearing action and this must not be neglected in determining

the width of the beam. The lowest limit permissible is indeed a matter not likely to be reached, but the same time it should be investigated. . . ."

This does not agree well with the previous paragraph, No. 6, in which the same author says: ". . . The neutral axis may be considered as a sort of equator of each section, since it passes through its center of gravity at right angle to the plane in which the bending takes place. It thus divides the sections into two portions, in one of which all the fibers which are parallel to the axis of the prism are subjected to a tension proportional to their distance from the neutral axis [the tension side of the section], while in the other portion the corresponding fibers are subjected to compression in a like proportion [the compression side of the section]. *It follows that fibers which are at equal distances from the neutral axis will be deformed to the same extent.* . . ."

Let us now consider the beam represented in Fig. 1. The neutral axis $x x_1$ passes through the center of gravity O of a section at $A B$. The bending force P acts perpendicularly to the length L. The part $A O$ is in tension and $O B$ in compression. After the load was applied the points A and B occupied the positions A_1 and B_1 respectively. [The distances $A A_1$ and $B B_1$ are of course greatly exaggerated in the figure, for the sake of demonstration.] A portion $b b_1$ of a layer of fibers situated at a distance a from B represents the amount by

which the compression shortened the layer $b b_2$ proportionally to its distance from the neutral axis. The dis-

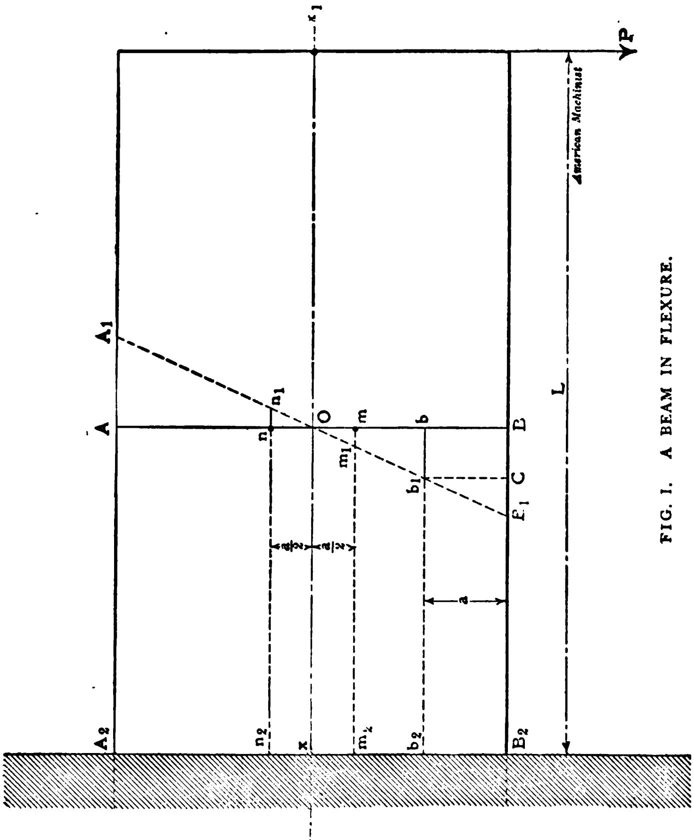

FIG. I. A BEAM IN FLEXURE.

placement of point B is $B_1c + Bc$ [line b_1c being parallel to AB]. Then B_1c represents the amount of sliding of the layer BB_2 over its neighbor bb_2. Two other points

n and m such that $no = mo = \frac{a}{2}$ occupy now the positions n_1 and m_1 respectively. The lines $n n_1$ and $m m_1$ are parallel, and the former represents the extension of the layer $n n_2$, or the amount of sliding of that layer over the neutral plane, which is assumed to be invariable, while the other is the amount of sliding of the compressed layer $m m_2$ over the same plane. Consequently the relative sliding of one layer over the other is $n n_1 + m m_1$. But angle $A O A_1 = B O B_1$ since tensions and compressions are proportional to their distances from the neutral axis, hence $n n_1 = m m_1$ and $n n_1 + m m_1 = B_1 c$. In other words, the longitudinal sliding or shearing and consequently the resistance to it are the same for all the layers of fibers, independently of their distances from the neutral axis, and are proportional to the leverage of the bending force P.

Mr. Jouravski remarked that if two equal rectangular prisms of length L, horizontal and fixed at the same end, are simply superposed, they will support together at their free end a weight P, which is only one-half of that which they could carry if prevented from sliding over one another, or if they had been cast together. This is expressed by the following equations:

For a solid prism,

$$P_1 = f \frac{b}{6L} \cdot h^2$$

For two superposed prisms,

$$P = f\frac{b}{6L} \cdot 2\left(\frac{h}{2}\right)^2$$

b is the width common to these beams, h is the sum of hights of the small beams and the hight of the solid prism.

Hence $$P_1 = 2P.$$

Instead of two beams having a total hight $= h$, suppose we have a number n of them, the hight of each prism will be $\frac{h}{n}$ and the load supported by the sum of the superposed beams will be:

$$P_n = f\frac{b}{6L} \cdot n\left(\frac{h}{n}\right)^2$$

Consequently $P_1 = n\,P_n$.

It is thus shown that the resistance to longitudinal shearing of the layers of fibers is proportional to the number of these layers, but not to their respective positions, as Mr. Jouravski stated.

We may also remark that, if the superposed prisms were fixed at both ends, the sliding would not take place as in the case just analyzed, and the system would behave as a solid of the same total rectangular section.

In the foregoing we have assumed that the bending was not great enough to cause the rupture of the beam, consequently it is quite admissible to conceive any trans-

verse section, *plane*, as well before as during the application of the load. But when permanent deformation preceding rupture is likely to occur, the sections do not remain plane, and equation (1) is then insufficient to express the relation between the exterior and the molecular forces. Mr. de Saint Venant in his Memoir on Flexure (1856) has conclusively proven that the shearing stresses, transversal and longitudinal, combine, in such a case, with the tensive and compressive stresses, the result being that any section originally plane becomes distorted in the form of a double curve, each half of which is a parabola of the third degree.

The safe resistance to the longitudinal shearing is:

$$f_l = \frac{3}{2} \frac{P}{bh} \quad . \tag{4}$$

as given by Jouravski, Belanger, Bresse, Reuleaux, etc. $= \frac{24}{5} \frac{PL}{bh^2}$

Professor Reuleaux makes $f_l = \frac{4}{5} f$. If equations (1) and (4) are combined and this value of f_l is introduced therein, we have;

$$\frac{5}{4} \times \frac{3}{2} \frac{P}{bh} = f = \frac{6 P x}{b h^2}$$

whence $h = \frac{16}{5} x = 3.2\, x$, quite a useless value for our problem.

Mr. L. Vigreux [Résistance des Matériaux] says: "If the material composing the beam is close-grained

$f_t = f$; but if of a fibrous texture, like wood, the value of f_t is different from f. No experiments have been made in order to determine the value of f_t, and for the present *we must solve the equations by choosing for* $\frac{h}{b}$ *the value adopted in practice.* . . ."

Therefrom we conclude that in practice the ordinary longitudinal shearing cannot be employed as a factor in the calculations, and that it may become dangerous only because of the bad choice of the point of application of the load, or on account of the defective manner in which the beam is fixed.

THE DEFLECTION OF THE BEAM.

This is indeed a most valuable auxiliary. It should be remarked, however, that the problem as stated contains no reference to it, and is thus in conformity with the majority of questions on strength of materials. In practice the deflection is not considered as much as it should be, and, while in many cases it deserves the first place in the calculations, it is too often neglected; attention being paid only to the strength of the parts. In machine tools, for instance, strength is secondary to

rigidity; but it is hardly necessary to state that, when rigidity is obtained in a solid, strength is generally ample and the factor of safety satisfactory.

The vibrations which are so objectionable in machine work should be minimized by making the parts very rigid—that is, by reducing as much as possible their deflection. It seems, then, that the general adoption of standards of rigidity would prove very beneficial.

When we consider the allowable stress f in bending we give it a value depending on the conditions under which the load is applied, viz., dead load, moving load, load causing alternative stresses of tension and compression, etc. Similarly, the deflection could be made a fraction of the length of the beam and given a value corresponding to the degree of rigidity required. Thus for general work, Weisbach ("Theoretical Mechanics," 8th edition, page 470) advocates making the deflection:

$$d = \frac{1}{500} \cdot L = nL$$

$L =$ length of beam in inches for d in inches.

It is only necessary to agree upon a few practical values of the quantity n to establish standards which, like the values of f, will not be binding but will simply indicate the limits suitable to certain conditions of work.

The deflection formula applicable to our problem is:

$$d = \frac{1}{m} \cdot \frac{PL^3}{3EI} \qquad (5)$$

where

E = modulus of elasticity of the material;

I = moment of inertia of the section resisting the greatest bending moment PL;

m = co-efficient depending upon the shape of the beam.

By comparing equations (1) and (5) we remark that they have some common factors:

$$(1) \quad . \; . \; . \; Px \text{ or } PL = f\frac{I}{a}$$

$$(5) \quad . \; . \; . \; d = \frac{1}{m} \cdot \frac{PL^3}{3EI}$$

By combining both we obtain:

$$d = \frac{1}{m} \cdot \frac{f}{3F} \cdot \frac{L^2}{a} \tag{6}$$

At last hère is something tangible! Knowing from the nature of the work expected the desired amount of rigidity of the parts, we may express d as a function of the length, and since a for a rectangular section is equal to $\frac{h}{2}$, we have:

$$d = nL = \frac{1}{m} \cdot \frac{f}{3F} \cdot \frac{L^2}{\left(\frac{h}{2}\right)}$$

whence:

$$h = \frac{2}{3} \cdot \frac{1}{m} \cdot \frac{f}{F} \cdot \frac{L}{n} \tag{7}$$

Thus the hight of the section is comparatively deter-

mined, for all the factors are known quantities. The co-efficient m is somewhat embarrassing to determine; it varies, as previously stated, according to the shape of the beam. If the beam is of rectangular section throughout its length, $m = 1$. When the hight h of the beam remains constant throughout the length the width b tapers gradually and becomes zero (theoretically) at the loaded end, and $m = \frac{2}{3}$. If the width b remains constant throughout, the length h diminishes to zero at the loaded end, and the longitudinal elevation of the beam is a surface limited by two parabolas or by a straight line and a parabola; then $m = \frac{1}{2}$. In these two last cases the beam is of uniform strength.

It is understood, of course, that this co-efficient can be determined for any solid by solving the general equation·

$$\frac{d^2 y}{dx^2} = EI,$$

which is given in text-books. But usually the beam is sketched roughly on the design so that an approximate value can be assumed for m. In beams of uniform strength when rigidity is required this value is rather nearer $\frac{2}{3}$ than $\frac{1}{2}$.

If the chosen section is rectangular, we have gone now as far towards solving the problem as the known formulas permit. It is true that judgment must be exercised in

choosing a value for m, but the limits of variation of this factor are known and the task is easy for an experienced man. The hight h once found, b is thereby determined for the maximum section at point of support and, a convenient number of sections being calculated at different points of the length, the shape of the beam is delineated. If the degree of rigidity required is very great and the beam very long, the hight may be so great that the beam, if designed according to formula (1), will be very thin and consequently apt to bend or warp laterally. Therefore another form of section must be adopted which will insure rigidity and stability as well as economy of material.

When the material composing the beam is isotropic the most economical is the box or I section. We can find the required hight of the maximum section by using formula (7), but what must be the width b, the thickness of the top and bottom flanges and of the web? And these questions once settled through "judgment," how can the shape of the beam be delineated without employing the tedious process of calculating a great number of sections at points along the length of the solid?

When so much trouble is encountered in dealing with the simplest case, it seems impossible to delineate satisfactorily the irregular forms of sections. We have now examined the most important points advanced by the

"Theory" and we have found that the consideration of the deflection of the beam is the only matter which has helped toward the solution of the problem; but our task is far from being completed and we are left in the midst of such difficulties that we can realize, at present, that many practical men have some excuse for sneering at such an incomplete "theory" and prefer to solve the question by "cut and try" methods. The timely arrival of our experimental data brings the proper light into the subject, establishes the Theory of the Flexure of Materials upon a firmer basis and helps to make it a science.

EXPERIMENTS.

A beam subjected to bending is divided into two distinct parts by an imaginary neutral plane passing along the longitudinal axis of the beam. Each part is subjected to stresses which vary directly, in any transverse section, as the normal distance from the neutral plane to any point of the section. Both maxima stresses are consequently located at the points farthest from the neutral plane.

The resistance of a solid to tensile stress is directly

proportional to the area of its section, but is independent of the form of this section.

Generally the resistance to compression is considered in two ways:

1. When the length of a solid is less than a certain multiple of the least dimension of its transverse section it is common in practice to admit that the resistance is directly proportional to the area of the section;

2. When the length is greater than the said multiple the solid tends to give way by buckling.

Since the length of a beam subjected to bending is usually very great as compared with its width of section, and as in extreme cases it has been observed that thin beams gave way by buckling or warping laterally in the compression part, it seems that this latter should not only be given the necessary area to resist the compression, but should also be provided against buckling. In fact, the whole beam is something like the combination of one rod and one column. Why not treat the rod for tension and the column for buckling? This interpretation once admitted numerous facts gathered from records of experiments on bending came forth to strengthen it. We see, for instance, in Navier's Work on "Resistance of Solids" (revised by de Saint Venant, page 113, Vol. I) that Mr. Ferdinand Zores made, in 1850, some experiments upon rolled beams. Mr. Zores remarked that symmetrical I-beams supported at the ends and loaded

at the middle always gave way by the buckling of the compression part, while the tension part was not altered. The beams were afterwards made with a reinforced compression part, with a view to remedy this defect; their resistance was somewhat increased, but they buckled the same as before. It was then remarked that the lateral bending was very similar to that of columns loaded on ends. General Morin also cites some similar cases in his "Résistance des Matériaux."

Mr. de Saint Venant almost pointed out the right solution of the question by stating (page 139 of Navier's work) that the resistance of a beam to buckling depends upon the ratio $\frac{I_1}{I}$ of the two principal moments of inertia of its section. But his subsequent analysis is incomplete because he considers only the section of the beam and does not take the length into account.

It remains to prove the correctness of the interpretation just advanced. As it will be seen presently, the experiments undertaken prove it very clearly.

Of all materials available for making tests, wood possesses so many good qualities that it is usually preferred to any other. Thus, I employed in my experiments strips of white pine carefully selected and free of imperfections. The modulus of elasticity of each was ascertained by the ordinary method: a strip of rectangular section being placed horizontally upon two supports

and loaded at the middle with a known weight, the deflection or vertical displacement of the center of the strip due to the load was measured, and, all the symbols being known, the symbols were replaced by their values in the general formula, which was then solved for E, the modulus of elasticity. For a beam of constant section throughout supported at the ends and bearing a uniformly distributed load P_1 and a single load P at center the deflection is:

$$d = \frac{L^3}{48\,EI} \cdot \left(\frac{5}{8}\,P_1 + P\right) \tag{8}$$

This deflection is composed of that d_1 due to the uniformly distributed load P_1, which is here equal to the weight of the beam, plus that d_2 due to the single load P. The formula can be written thus:

$$d = d_1 + d_2 = \frac{5}{8} \cdot \frac{P_1\,L^3}{48\,EI} + \frac{P\,L^3}{48\,EI}$$

The deflection d_1 itself need not be measured, but if d_2 is ascertained we have:

$$d_1 = \frac{P\,L^3}{48\,EI}$$

whence

$$E = \frac{P\,L^3}{48\,d_2\,I}$$

I is the moment of inertia of the section of the beam referred to an axis passing through the center of gravity

of the section and perpendicular to the direction of the load.

The plan adopted for tests was as follows: A beam of rectangular section of base $= b$ and hight $= h$ was placed horizontally upon supports and loaded at center. In every case h was several times larger than b. *The load was slowly increased until the beam suddenly bent or warped laterally, and becoming unstable would have fallen from its supports—had not preventive means been provided—but without breaking at any place.* This test was repeated twice more and the maximum load recorded. The distance between supports being shortened by one inch on each side of the center (totally by 2 inches) a new maximum load was recorded, and so on until the strip gave way by breaking across at the point of application of the load (where the bending moment was greatest). This gave the variation of the load corresponding to that of the length. Then another strip was tested, it having the same hight h as the preceding one, but a different thickness b. This strip was treated in the same manner as the first one, and, the results being compared, the law governing the variation of the load for different thicknesses was found. Finally, a strip of the same thickness b as the first, but having a different hight h, was tested similarly to the others. The results again compared with the previous ones, gave the law governing the variations of the load for different hights.

The results observed in the first three tests were repeated with a remarkable constancy in all the others in such a manner that, for a number of tests, the loads were calculated in advance, and the subsequent experiments proved the correctness of the provisions. The laws found are:

1. *The load is inversely proportional to the square of the length* L *between supports.* Thus, if P and P_1 are the different loads applied at the center of one beam, the distances between supports being respectively L and L_1, we have:

$$\frac{P}{P_1} = \frac{L_1^2}{L^2} \qquad (9)$$

2. *The load is directly proportional to the cube of the thickness* b *of the beam.* Thus two beams of the same length L between supports and of the same hight h, but of different thicknesses b and b_1 will bend laterally or warp under the respective loads P and P_1, and we have:

$$\frac{P}{P_1} = \frac{b_1^3}{b_1^3} \qquad (10)$$

3. *The load is directly proportional to the hight* h *of the beam.* Two beams of same thickness b and same length L between supports but having different hights h and h_1 will warp under the respective loads P and P_1, and we have:

$$\frac{P}{P_1} = \frac{h}{h_1} \qquad (11)$$

The three laws combined into one are represented by the equation:

$$\frac{P}{P_1} = \left(\frac{L_1^2}{b_1^3 h_1}\right) \cdot \left(\frac{b^3 h}{L^2}\right) \qquad (12)$$

If we multiply and divide the second member of the equation by 12, its value is not altered and it may be written:

$$\frac{P}{P_1} = \left(\frac{12}{b_1^3 h_1} \cdot L_1^2\right) \left(\frac{b^3 h}{12} \cdot \frac{1}{L^2}\right) \qquad (13)$$

But $\frac{b^3 h}{12}$ is the moment of inertia of a section referred to an axis parallel to the hight h and passing through the center of gravity of the section. The same remark applies to $\frac{b_1^3 h_1}{12}$, therefore:

$$\frac{P}{P_1} = \frac{L_1^2}{I_1} \cdot \frac{I}{L^2} \qquad (14)$$

Which shows that the load is directly proportional to the least moment of inertia of the section and inversely proportional to the square of the length between supports. For two beams of uniform sections $b\,h$ and $b.\,h_1$ we have also:

$$\frac{P\,L^2}{P_1\,L_1^2} = \frac{I}{I_1}$$

and for one beam subjected to different loads P and P_1 at the respective distances L and L_1

$$\frac{PL^2}{P_1L_1^2} = \frac{I}{I} = 1,$$

whence

$$PL^2 = P_1L_1^2 = \text{constant}.$$

Consequently, if for a certain length L it is found that a load P causes the beam to give way laterally, the expression just established permits to determine in advance the deflecting load P_1 corresponding to any length L_1, or $P_1 = \frac{PL^2}{L_1^2}$.

This deduction has been fully confirmed by the experiments. A cursory analysis brings out the very important fact that, for any given length, there is a corresponding load which causes the lateral bending. When L is infinitely great (neglecting, of course, the weight of the beam) P is infinitely small, and *vice versa*; but the product PL^2 is always a constant, and that, irrespective of the moment of inertia of the section or of the shape of this latter.

Numerous tests of wooden beams of I or T sections have been made by the author, and they proved beyond a doubt the truth of the new laws.

It is, of course, impossible to prove the expression $PL^2 =$ constant when L is very small, because P becomes so great that the beam would be crushed at the points of application of the load or of support, and the real object of the experiment could not be attained.

There is also a certain value of P corresponding to which the resulting bending moment $\frac{PL}{4}$ just balances the moment of the molecular forces developed in the most strained section of the beam (at the point of application of the load). Then a very slight increase of the load would cause the beam to give way by breaking across at a place situated at or very near the middle of the lengths between supports. From this it follows that when a beam is loaded in such a manner that

$$PL = 4f\frac{I}{a} = \frac{(PL^2 = \text{constant})}{L}$$

it is just as liable to buckle as to break across.

In this equation f is the breaking strength of the material of the beam and $\frac{I}{a}$ is the moment of resistance of the transverse section.

When the beam starts to buckle it assumes at once a characteristic shape which is accentuated as the load is gradually increased. The well-known parabolic curve due to the bending lies in a vertical plane, while that due to buckling, as viewed from above is S-shaped, as represented in Fig. 2. This peculiar form is very misleading, and the author, noticing it in many of his experiments, came to the conclusion that it was a double curve, and he was at a loss to explain its non-appearance

as such when the length of the beam between supports was very small. However, after very careful trials, it was found that in short beams requiring a heavy load to cause the buckling, the rigidity was very great and the lateral curve assumed was like an arc of circle, whereas in long beams, easily buckled, the rigidity was very small and the arc-like curve assumed at the beginning of the buckling was quickly distorted into the shape shown in Fig. 2, as the load increased appreciably Moreover, with beams of I and T sections, the S curve was more difficult to obtain owing to the greater rigidity.

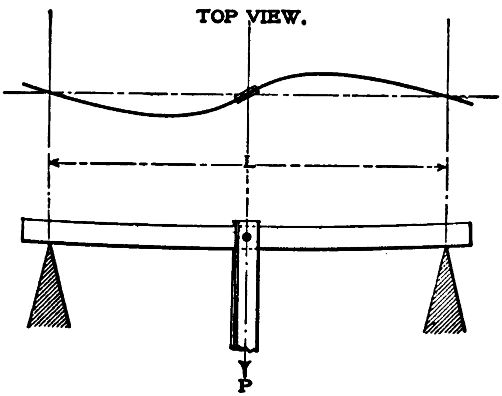

FIG. 2. FAILURE OF A BEAM BY BUCKLING.

The contraflexure at the center of the beam is caused principally, as will be seen later, by the guiding due to the suspension strips fastened to the beam and through which the load is transmitted. Thus, when this double

curve manifests itself, it is still possible to increase the load by about one-half before the elastic equilibrium of the beam is destroyed, because the new shape of the beam increases its static equilibrium. This explains why the recorded loads for the plain rectangular sectioned beams are so much in excess, relatively, of those applied on the I beams. (The exact manner in which all the experiments were conducted will be fully explained later.)

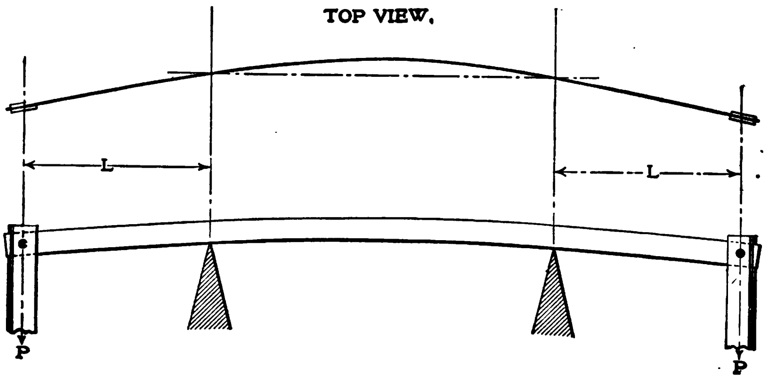

FIG. 3. FAILURE OF AN OVERHUNG BEAM.

In another series of experiments in which a bar was supported horizontally at two points equally distant from the ends, and equal weights were applied at the ends, it gave way also by buckling, but the resultant curve, lying horizontally, was in the form of an arc of circle, as shown in Fig. 3. Here we have a single curve when the ends of the beam are free to move under the

action of the load, while in the first case the ends were held on the supports by the friction due to the load. The deformations of the beam, as stated above, as well as the consideration of formula (14) modified as follows:

$$P = K\frac{I}{L^2} = \frac{\text{constant} \times I}{L^2}, \tag{15}$$

where $K = \frac{P_1 L_1^2}{I_1}$, point to a similarity of effects between those produced in a beam by direct bending and those due to an axial thrust causing lateral bending (case of a

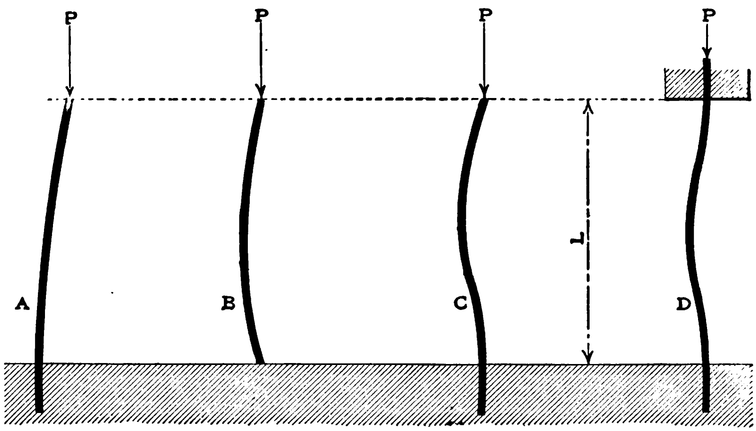

FIG. 4. FAILURE OF STRUTS.

column loaded on end). This lateral bending distorts a strut differently according to the manner in which the strut is supported. For instance, if a solid is fixed at one end and loaded axially at the other, the resulting

curve has half a bend, while that which results when the solid is only supported instead of fixed at one end has one. In Fig. 4 are represented struts supported or held in various manners, and if the curve assumed at *B* is taken as a standard of comparison we see that at *A* the curve has ½ bend, at *C* the curve has 1½ bends, and at *D* the curve has 2 bends.

These curves analyzed by Euler are said to be sinus-soidal arcs, though it can be proved that they are only approximately such; their theory is exposed in advanced treatises on Applied Mechanics. Professor W. C. Unwin treats of the compression of long bars in a clear and concise manner in his "Elements of Machine Design," Vol. 1, section 38, from which the author begs to quote the following: ". . . If, however, the bar is of great length it gives way ultimately under the action of a thrust by lateral bending; the stress at the section where the fracture occurs being a compound stress, due both to the longitudinal pressure and the curvature of the bar. Rules for the ultimate resistance of long bars to compression were first obtained theoretically by Euler and experimentally by Professor Hodgkinson. Hodgkinson's formulas have been generally used in this country (England) in designing compression bars. These rules are inconvenient in form, and they can only be extended to many cases of common occurrence by theoretical assumptions, which are only approximately true.

An expression more convenient in form than Hodgkinson's was proposed by Tredgold, revised by Gordon, and afterwards modified by Rankine, so as to be applicable to bars of all forms of section. The reasoning on which this rule is based is, however, not satisfactory. All these rules are intended to give the ultimate strength of the bars, and in applying them it is necessary to divide the resistance thus calculated by an arbitrary factor of safety. Actual compression bars are not intended to be loaded beyond their elastic limit, and hence it may be urged, with reason, *that the theoretical formulas of Euler, which Hodgkinson discarded as not agreeing with his experiments on ultimate strength, are more strictly applicable to the circumstances in which long compression bars are used than Hodgkinson's rules.* They are simpler and include all cases. Euler's rules assume the bar to be initially straight and homogeneous, the load axial and the elasticity to be unimpaired with the greatest load. In that case no increase of the load would directly cause bending, but a point is reached at which the equilibrium of the bar becomes unstable. With less loads the bar, if slightly bent temporarily, will restore itself to straightness. With greater loads, if any flexure is produced, however slight, the bar will not restore itself to straightness, but the bending will increase under the action of the load till the bar breaks. . . . ."

The general formula due to Euler,

$$P_0 = \frac{n^2 \pi^2 E I_0}{L_0^2}, \tag{16}$$

shows that when a straight bar of length L_0 supported or held at one end is loaded axially at the other, its equilibrium becomes unstable when the value of the load P_0 is equal to a constant factor $(\pi^2 E)$ multiplied by least moment of inertia I_0 of the section of the bar, and by the square of the number of bends in the curve assumed by the bar, and divided by the square of the length L_0.

If for a bar of uniform section we vary the load P_0 or the length L_0 without changing the mode of application of the load, the quantity $(n^2 \pi^2 E)$ becomes a constant and we have the formula:

$$P_6 = \frac{\text{constant} \times I}{L_0^2}, \tag{17}$$

which is identical with our formula (15) found from the experiments on flexure. The constant may not be the same in both cases, since the curve produced by compression is a sinussoid situated in one plane, whereas that due to flexure is the combination of a cubic parabola in the plane of flexure with what may be assumed to be a sinussoid in another plane normal to the former, the resultant being a curve in space. Moreover, we must bear in mind that the load is applied axially in one case

and transversally in the other. Consequently, the terms $(n^2 \pi^2)$ may have to be modified by a co-efficient (Φ) whose nature we will endeavor to determine.

At this stage, when only too ready to launch into purely mathematical analysis based upon the unexpectedly successful results of a relatively small number of experiments, the writer was halted by the following consideration: Since the laws of the elastic equilibrium of a beam subjected to flexure have such a striking similarity to the theoretical laws of Euler governing the equilibrium of a long rod loaded axially, it would be logical to ascertain experimentally at once the degree of similarity of the laws. Consequently another series of tests was undertaken in which more care was exercised than in the previous one. Each specimen (a strip of rectangular section) was held at one end and carried at the other a gradually increased load. When the strip buckled, the load was recorded and removed; then the length being augmented by one inch, a new load was applied until the strip buckled anew and so on. In all cases the load acted vertically while the specimen occupied successively the positions:

1. Horizontal.
2. Inclined at 30° with the horizon.
3. Inclined at 45° with the horizon.
4. Inclined at 60° with the horizon.
5. Vertical, the load acting axially.

Fig. 5 represents the apparatus employed for testing the specimen in four positions. The strip is placed between two wooden plates A and B, where it rests upon two pins C and C_1. The axis of pin C is in the plane formed by the two outward ends of the boards, so that the length L of the specimen may be measured from the center of the pin to the center M of suspension of the load. The boards are firmly pressed together by means of the clamps D and E. Consequently the beam may be truly considered as being *firmly held at one end and loaded at the other*. A large pin F passes through the two boards and extending on each side rests upon the supports N and N_1. The boards are cut at various points in such a manner that if one of these is brought upon a horizontal plane tangent to the lower side of the pin F, the specimen is correspondingly held in one of the four positions described above. Thus, point G corresponds to the horizontal position, point H to the 30-degree position, point K to the 45-degree position, and point L to the 60-degree position.

The point selected rests upon a strong steel bar V. From the pin F is suspended a rod T holding a counterpoise W, which preserves the equilibrium of the system when the load is applied.

Two thin strips of wood U and U_1, firmly bound together at the lower end by a small bolt and nut, are secured at the top—one on each side of the specimen—

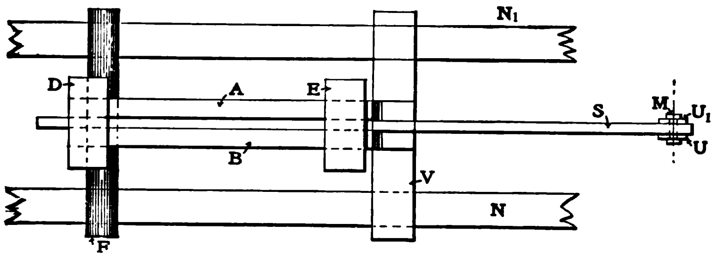

FIG. 5. TOP VIEW.

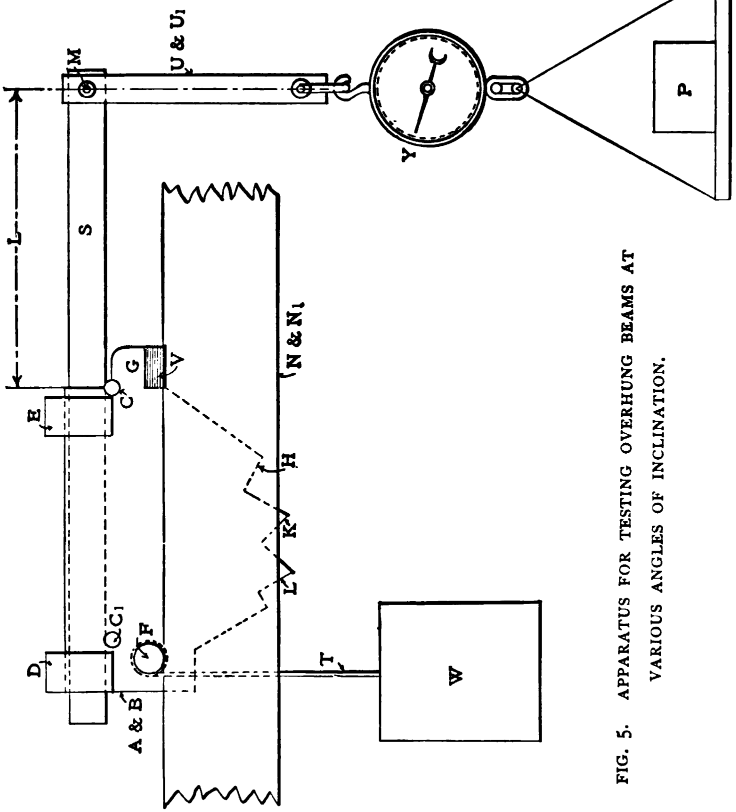

FIG. 5. APPARATUS FOR TESTING OVERHUNG BEAMS AT VARIOUS ANGLES OF INCLINATION.

by a small bolt passing through the center of suspension *M*, and are lightly kept together by the pressure of the nut. To the lower bolt is attached a spring scale carrying a board upon which is placed the load *P*, composed of blocks of iron. The total load supported by the beam is the sum of the weights of the strips, scale, board and blocks.

The mode of proceeding is very simple. A length *L* being selected for a given beam secured in the horizontal position, a load is gradually applied until buckling occurs. The load is recorded and removed. Then bar *V* is withdrawn and placed under point *H* just brought into the horizontal position by rotating the system about the rod *F* as an axis. The beam is thus inclined at an angle of 30 degrees with the horizon. A new load is tried, recorded and removed, and the system is again rotated with point *K* on the horizontal plane, thus inclining the beam at 45 degrees. And so on until a load is recorded respectively for each of the four positions. Then the system is again placed horizontally, the clamps *D* and *E* are loosened, the length *L* augmented by one inch, and the clamps being fastened anew another cycle of operations is gone through, and so on until the length becoming too great the bar is not rigid enough to insure the necessary exactness of the tests.

The same apparatus cannot be used for experiment-

ing in the fifth position, because in order that the test be exact it is necessary: First, that the bar be held firmly at the base; second, that the load be applied at the top, and that its direction coincide with the axis of of the bar; third, that the bending or buckling of the bar be not hindered in any manner. This last condition is very important, as will be seen later, and cannot be complied with with the first apparatus, because the strips U and U_1 being placed in the direction of the probable deflection of the bar, would prevent this latter from bending freely.

Consequently, the apparatus represented in Fig. 6 was devised. To the supporting beams N and N_1 are fastened the wooden blocks A and B, respectively. The specimen S is placed between these latter and adjusted vertically, then the clamps D and E are tightened and the bar is thus firmly held. The clamps press against two steel bars FF_1, which distribute the pressure uniformly. The load is applied by means of a frame consisting of two cross plates MM_1 attached to the rods U and U_1. Plate M is knife-shaped at the base, but the edge is very dull, The scale Y attached to M_1 sustains, as before, the board and weights P. The load consists of the sum of the weights of the frame, scale, board and weighing blocks. The length of the bar is measured from its top to the upper face of the blocks A and B.

The bar being firmly held, the load is gradually

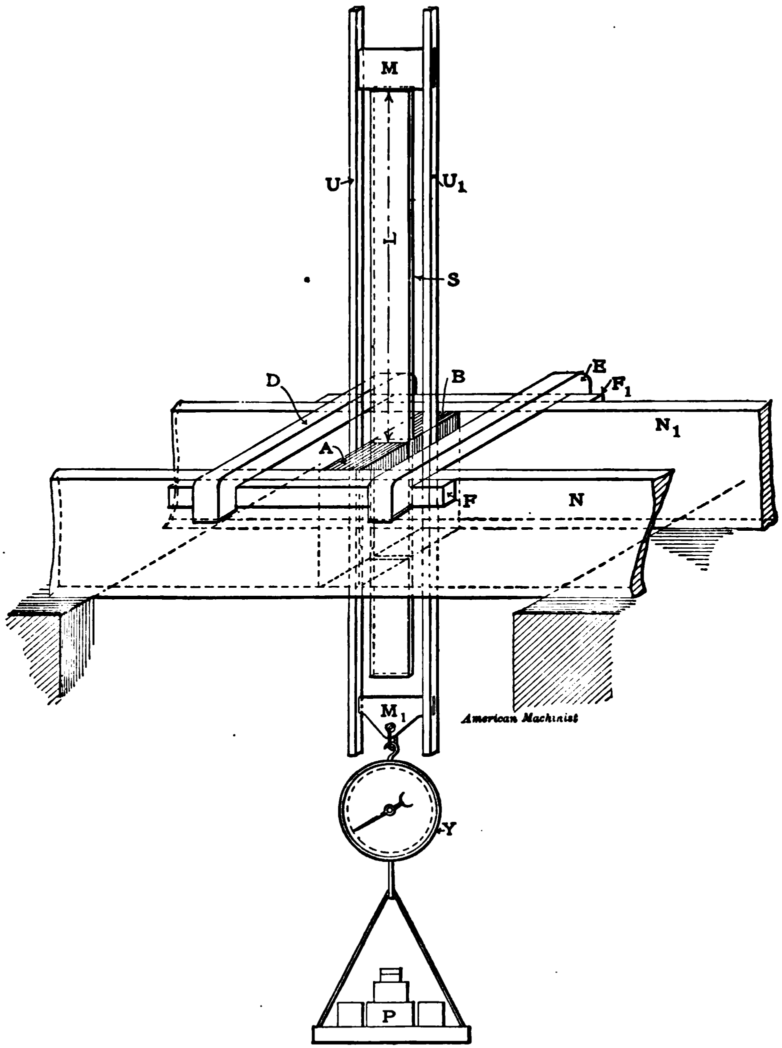

FIG. 6. APPARATUS FOR TESTING STRUTS LOADED AXIALLY.

applied. Although the weighing blocks are placed upon the board with extreme care, the loading frame is not perfectly still; in fact, it oscillates like a pendulum and consequently communicates its motion to the top of the bar. Thus the bar vibrates gently and uniformly on each side of the axis, until about seven-eighths of the total load have been placed, at which time it becomes lazy; in other words, when during the period of one vibration the axis is vertical the bar seems to bend on one side in a shorter time than it takes to regain the vertical position. With a further increase of the load the bar finally fails to oscillate and remains slightly bent to one side. With less load it oscillates again, but with a further increase the bar falls forward with great velocity and would break were it not stopped in time. The load corresponding to which the bar failed to oscillate is recorded as being the maximum that the bar can hold in equilibrium. The weighing blocks are then removed, as well as the frame, and the clamps being loosened the bar is raised one inch and fastened anew. The cycle of operations is again gone through, and so on until the length of the bar becomes too great to permit an accurate test.

Some difficulty was found in placing the cross bar exactly on the center of gravity of the upper face of the beam. If M is not rightly placed the load does not act axially on the bar and consequently near the limit the

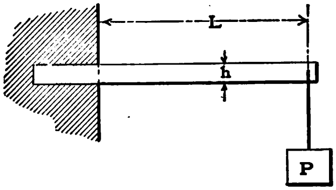

Fig. A

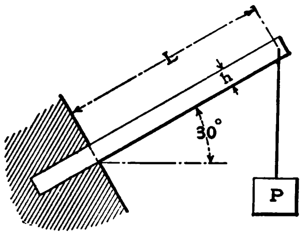

Fig. B

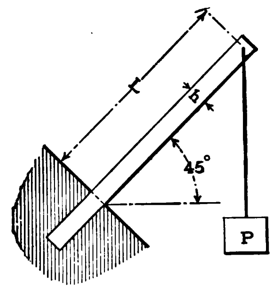

Fig. C

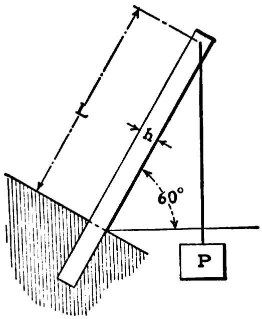

Fig. D

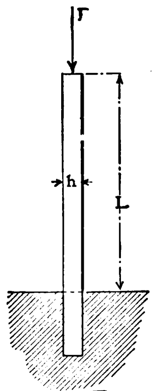

Fig. E

REFERENCE FIGURES FOR THE TABLE OF TESTS.

beam bends toward one side only. The cross bar should be shifted a little, until a position is attained at which the beam bends indifferently to one side or the other. The carrying capacity is thus increased, and is maximum when the load acts axially.

The results of experiments performed on nine bars of various sizes, but all of rectangular section, are shown in the following tables. In Table I we have the dimensions of the bars as well as the moments of inertia and the

No.	b	h	I	I_1	E
1	1/8″	1″	0.01042	0.00016276	1265000
2	3/16″	1″	0.015625	0.00054932	1226400
3	1/4″	1″	0.02083	0.0013025	1305150
4	1/8″	1½″	0.035156	0.00024414	1292450
5	3/16″	1½″	0.052734	0.00082397	1361750
6	1/4″	1½″	0.070312	0.0019531	1278750
7	1/8″	2″	0.08333	0.00032552	999455
8	3/16″	2″	0.125	0.00109864	1488200
9	1/4″	2″	0.1666	0.002605	1315050

TABLE I. DIMENSIONS, MOMENTS OF INERTIA AND MODULII OF ELASTICITY OF BARS TESTED.

No. 5		No. 6		No. 7		No. 8		No. 9	
P	PL^2	P	PL^2	P	PL^2	P	PL^2	P	PL^2
33.5	3350.								
27.5	3327.5								
23.	3312.			9.75	1404.				
20.75	3506.75			8.	1352.				
17.25	3381.	36.5	7154.	7.	1372.				
15.	3375.	31.75	7143.75	6.25	1406.25	21.	4725.		
13.25	3392.	27.75	7104.	5.5	1408.	17.75	4544.		
11.5	3323.	24.5	7080.5	4.625	1336.625	15.75	4551.75		
		21.75	7047.	4.125	1336.5	14.50	4698.	31.	10044.
		19.75	7129.75	3.625	1303.625	13.25	4783.25	27.25	9837.25
		17.75	7100.	3.375	1350.	11.75	4700.	24.75	9900.
		16.	7056.			10.50	4630.5	22.50	9922.5
		14.5	7018.			10.	4840.	20.50	9922.
						8.75	4628.75	19.	10051.
						8.	4608.	17.	9792.
								15.75	9843.75
								14.50	9802.
	3370.96		7092.8		1361.55		4670.95		9901.61

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ALLY AT ONE END AND LOADED AT THE OTHER, AS IN FIG. A.

No. 5		No. 6		No. 7		No. 8		No. 9	
P	PL^2	P	PL^2	P	PL^2	P	PL^2	P	PL^2
9.5	2950.								
5.5	3085.5								
2.5	3240.			8.25	1188.				
3.75	3168.75			7.25	1225.25				
.25	3185.	32.	6272.	6.	1176.				
3.5	3037.5	27.25	6131.25	5.125	1153.125	19.5	4387.5		
2.25	3136.	23.	5888.	4.5	1152.	16.5	4224.		
1.25	3251.25	21.5	6213.5	4.	1156.	14.75	4262.75		
		20.	6480.	3.5	1134.	13.75	4455.	27.	8748.
		18.	6498.	3.125	1128.125	12.5	4512.5	23.75	8573.75
		15.25	6500.	2.75	1100.	11.25	4500.	21.	8400.
		13.75	6063.75			9.75	4209.75	19.25	8489.25
		12.75	6171.			8.75	4235.	18.	8712.
						8.	4232.	16.5	8728.5
								15.	8640.
								13.75	8593.75
								12.75	8619.
	3131.75		6246.38		1156.94		4345.38		8611.56

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D AT AN ANGLE OF 30 DEGREES, AS IN FIG. B.

See Figure E

No. 2		No. 3		No. 5		No. 6		No. 8		No. 9	
P	PL^2	P	PL^2	P	PL^2	P	PL^2	P	PL^2	P	PL^2
				26.75	26.75						
				23.375	2828.875						
25	1764.			19.	2646.						
5	1732.25			16.125	2725.125						
5	1715.			13.75	2695.	30.75	6036.				
5	1687.5			12.	2700.	27.	6075.	18.	4050.		
25	1696.			10.625	2720.	23.75	6080.	16.5	4224.		
	1734.	14.75	4262.75	9.625	2781.625	21.25	6141.25	14.5	4190.		
25	1660.5	13.25	4293.			19.	6156.	12.75	4131.	26.25	8505.
5	1714.75	11.75	4241.75			Broken		11.5	4151.	23.5	8483.5
25	1700.	10.75	4300.					10.25	4100.	21.5	8600.
	1764.	9.5	4189.5					9.25	4079.25	19.625	8654.625
		8.625	4174.5					8.375	4053.5	17.875	8651.5
		8.	4232.					7.75	4099.75	16.25	8506.25
		7.625	4392.					7.	4032.	15.	8645.
										13.875	8671.875
										12.75	8619.
	1716.8		4260.68		2721.39		6097.65		4111.05		8602.97
	1662.3		4186.7		2768.4		6162.2		4034.2		8449.5

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5 AND 60 DEGREES AND VERTICALLY, AS IN FIGS. C, D AND E,

modulii of elasticity. In Table 2 are given the results of the tests on bars held horizontally at one end and loaded at the other. The product PL^2 of the load by the square of the length being almost a constant quantity, the first law is verified in all the tests, but the second and third laws are far from being as clearly established.

It would seem, so far, that the laws need to be modified in a fashion similar to that employed by Professor Hodgkinson in his experiments on columns.

The tests with the bars inclined at an angle of 30 degrees with the horizontal are recorded in Table 3. Here again the first law is clearly verified, but the other two are not conclusively proven.

In Table 4 are recorded the tests for bars Nos. 1, 2, 3, and 6 inclined at 45 degrees, and for bar No. 3 inclined at 60 degrees. Great difficulty was experienced in these last tests, in determining the exact loads causing the buckling; in fact, with the 60-degree inclination, the results were so erratic that they did not seem to be governed by any definite law. Each trial was a failure and I felt very much discouraged. But after a careful investigation it was found that the thin wooden strips U and U_1, Fig. 5, which were fastened to the specimen by a bolt passing through the center of suspension M, although very lightly pressed against the bar, caused this latter's extremity to remain vertical, thus preventing

to a certain extent the bar from assuming the curvature generally produced by the buckling.

Since the horizontal and 30-degree tests appeared to be sufficiently accurate, they alone were entirely carried out. The 45-degree tests were not quite completed and those at 60 degrees were abandoned for the time being. Then the bars were tested in the vertical position, the results also being shown in Table 4. Here the three laws are entirely verified, and the results, on being compared, are almost identical with those indicated by the formula of Euler, viz.:

$$P L^2 = \frac{\pi^2}{4} E I.$$

The ratios between actual and theoretical results are shown at the bottom of the tables. Consequently, it must be admitted that Euler's formula is correct, since it can be proved by such simple experiments as those just described. It is worthy of remark that the vertical tests were made under perfect conditions: *the bar being firmly held at the base and the load made to act axially in such a manner as not to hinder in any way the bar from assuming the natural curvature due to buckling.*

In order to investigate the cause of the failure of the other tests, a bar was placed at 60 degrees, and loaded until it buckled and fell to one side, when it rested against support *Z*, Fig. 7. Then an arm *a* was clamped

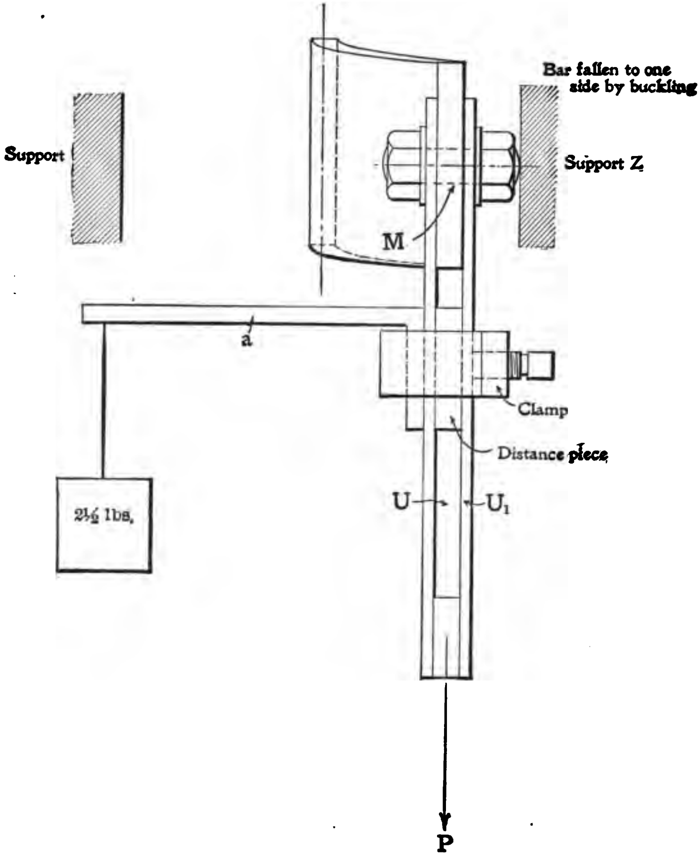


FIG. 7. THE ACTION OF AN ECCENTRIC LOAD.

to the strips U and U_1 near the top, and gradually loaded until the bar straightened slowly, and finally bending abruptly fell to the other side against a support, being

thus prevented from breaking. This bar was No. 3, having for dimensions $b = \frac{1}{4}$ inch, $h = 1$ inch. For a length $L = 18$ inches, it buckled under a load of $14\frac{3}{4}$ pounds, with the suspension bolt at M loosely fitted.

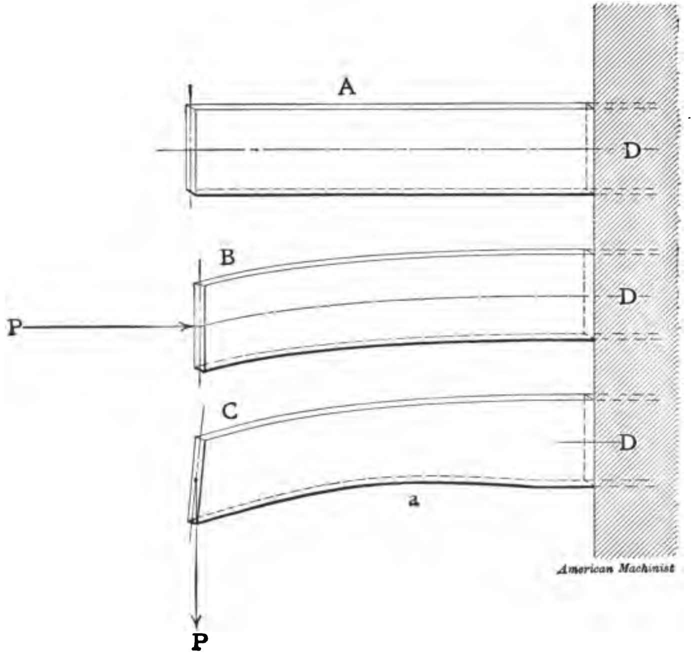


FIG. 8. FAILURE OF A BAR UNDER COMPRESSION AND BENDING FORCES.

This bolt being very firmly tightened, the bar straightened a little and $2\frac{1}{4}$ pounds were added before buckling occurred anew. Then the arm a being adjusted in place and loaded with about $2\frac{1}{4}$ pounds, the bar again straight-

ened slowly and with a slight increase of the load it fell on the other side against the support. This little experiment, repeated on several bars, proved conclusively that a bar can be made to bear a greater load if its loaded end is prevented from turning about its axis than it would carry were this end free to oscillate. This eccentric load simply amplified the guiding action of the strips U and U_1.

Fig. 8 represents at A a straight bar held at the end D. At B the same bar under axial compression is bent laterally, the curve assumed by this axis being a sinussoid combined in the same horizontal plane as the originally straight axis. This curve is the first of the nine kinds of plane elastic curves found by Jacques Bernouilli (A. D. 1694) and analyzed by Euler. It is seen that the vertical axis of the loaded end remains parallel to that of the bar in a state of rest, At C the bar is subjected to a bending effort and buckles in the compression part. Just before buckling occurred the bar was longitudinally straight as at A, but vertically bent, since the bending force necessarily caused a certain deflection. Consequently, the neutral plane of the bar affected a parabolic curve. With an increase of the load the compression part buckled and was curved, as shown at a. From the fact that the bar is thrown to one side it is easily seen that the two lower edges are no longer equally strained. The right edge is under a greater com-

pression and consequently shortened, but the left is somewhat relieved and therefore relatively elongated. The curve assumed by the upper edges is very much like that at B, and has not the abruptness seen at a. *It results from this that the major axis of the loaded end is no more vertical, and the bar is actually twisted.* Therefore any force which can prevent this twisting must necessarily increase the carrying capacity of the bar, or, in other words, must insure to a greater degree the elastic equilibrium of the solid.

It remains now to find the true laws governing the elastic equilibrium of a solid loaded transversely at one end and free to assume any consequent curvature. If at C, Fig. 8, the load is applied on the upper face of the bar, there will result an increased twisting and the bar will carry less load. If the load is hung from the lower face, the twisting will be lessened and the bar will be guided vertically, as with the strips U and U_1, thus increasing the carrying capacity.

Consequently, the point of suspension of the load at the end of the bar must be at the center of gravity of the section and must remain there. This condition can be fulfilled by employing the device illustrated by Fig. 9, where A is the end of a bar through which passes a rod B, made fast to the bar by means of the nuts and washers C and D. The ends of the rods are hook-shaped, and therefrom are suspended two rods E and F support-

ing the beam *G*. The centers of the hooks of rod *B* are equidistant from the center of gravity of the section

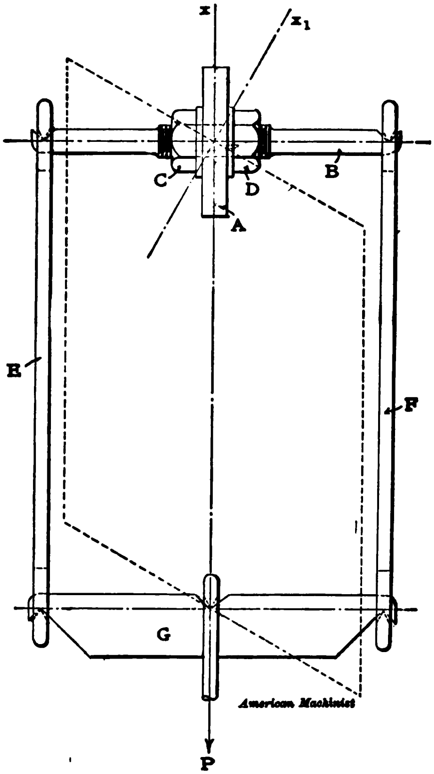


FIG. 9. METHOD OF SUSPENDING LOAD GIVING BAR ENTIRE FREEDOM TO BUCKLE.

A; the load *P* is applied at middle of beam *G*, and rods *E* and *F* are parallel. The whole system can oscillate freely, the points of suspension being all knife-edged.

The direction of the load passes therefore through the center of gravity of the section, and does so regardless of the inclination of the axis x of the bar—as, for instance, when the bar being twisted this axis occupies the position x_1, Fig. 9.

With this apparatus the bars were again tested in the four positions previously described, and, curiously enough, *were found to give way by buckling under exactly the same loads that caused the buckling in the fifth or vertical position.* Thus, bar No. 5, with a length of 10 inches, gave way in any of the five positions under a load of $26\frac{3}{4}$ pounds. The load was not in the full sense of the word exactly $26\frac{3}{4}$ pounds, but the variation was less than $\frac{3}{8}$ of a pound one way or the other. This variation is, of course, insignificant and well within the permissible limits. All the bending tests were successful, but the thin bars Nos. 1, 4 and 7, being only $\frac{1}{8}$ inch thick, offered much difficulty in the vertical position, because the load could not be applied axially.

We must now find the reason why a bar of iron of given length buckles under a certain load, as well when held vertically as horizontally, or in any intermediate position between these two.

The problem of the strength of columns properly belongs to the domain of higher mathematics. It has been solved by Euler, Lagrange, Poisson and several other great mathematicians. It is consequently impos-

sible to present an elementary solution of it here, and we must admit the correctness of Euler's formula since experiments prove it; we must also admit, for similar reasons, that the same laws govern the equilibrium of a horizontal beam subjected to a bending effort. Then we can easily explain the other cases. In Fig. 10 a

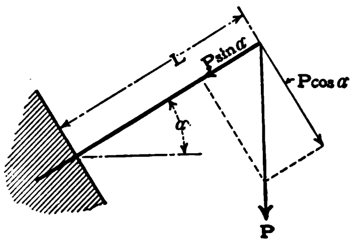


FIG. 10.

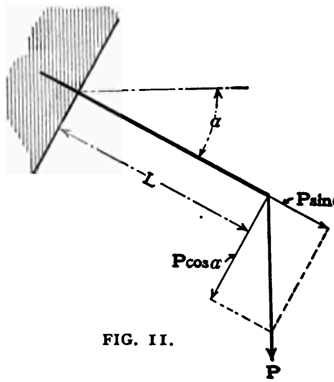


FIG. 11.

INCLINATION OF BAR ABOVE AND BELOW THE HORIZONTAL.

beam L firmly held at one end is inclined at an angle α from the horizontal and sustains a load P applied at the other end. The force P can be resolved into two others: One (P cos. α) acting perpendicularly to the length L produces a bending effort whose moment is P cos. $\alpha \,.\, L$; the other P sin. α, is a force of compression acting axially on the solid. It is very important to remark that $(P \cos.\ \alpha)\ L = P\ (L \cos.\ \alpha)$. The first member of

this equation is the product of the force P cos. α by the length L, whereas the second member is the product of the force P by the length L cos α. This second is more proper than the first, judging by the definition of the term *moment* of a force. Thus a moment is the product of a force by its lever arm. Here the force is P, the arm is the projection L cos. α. If the axial force alone was sufficient to cause the buckling of the prism, we would have

$$(P \sin.\ \alpha)\ L^2 = \text{constant};$$

but this expression is not correct, for we can also prove that

$$(P \sin.\ \alpha)\ L = P\ (L \sin.\ \alpha),$$

and the results would differ widely were we to introduce one or the other of these terms into the previous equation. But if the question is considered in another way we can make the results agree with the experiments. Thus, by assuming that the force P tends to compress a solid of length L sin. α, and to bend another of length L cos. α, we must have:

$$P\ (L \sin.\ \alpha)^2 = \text{constant}.$$

If the bending alone caused the buckling we would have:

$$P\ (L \cos.\ \alpha)^2 = \text{constant};$$

but, since the combination of the efforts caused the buckling, and since the load applied is equal to that

which buckled the same solid of length L in the vertical as in the horizontal position, it becomes evident that the efforts must be added. Hence

$$P\,(L\cos.\,\alpha)^2 + P\,(L\sin.\,\alpha)^2 = \text{constant}; \qquad (18)$$

or

$$P\,L^2\,(\sin.^2\,\alpha + \cos.^2\,\alpha) = \text{constant}.$$

But $\sin.^2\,\alpha + \cos.^2\,.\,\alpha = 1$, hence for any position between the vertical and the horizontal (above the latter) we have:

$$P\,L^2 = \text{constant}.$$

as found by experiment.

When the bar is placed below the horizontal, as in Fig. 11, the force P may again be resolved into two others, one ($P\cos.\,\alpha$) perpendicular to the axis of the bar, producing a bending effort whose moment is $P\cos.\,\alpha\,.\,L$; the other force ($P\sin.\,\alpha$) is axial and tensile. Since a tensile force cannot produce buckling, but, on the contrary, tends to prevent it, its effect is negative and, as in the present case, a bar nevertheless may buckle, it follows that it does so under the combined efforts of the components of force P. Therefore, we have again:

$$P\,(L\cos.\,\alpha)^2 + [-\,P\,(L\sin.\,\alpha)^2] = \text{constant},$$

or

$$P\,L^2\,(\cos.^2\,\alpha - \sin.^2\,\alpha) = \text{constant}. \qquad (19)$$

This shows clearly that the greater the angle α the less will be the tendency of a bar to buckle. Furthermore, this tendency will be maximum with the bar hori-

zontal, when sin. $\alpha = 0$, and minimum with $\alpha = 45$ degrees, when sin. $\alpha =$ cos. α and P becomes ∞. With a still greater angle, the tensile force being superior to the bending, no buckling can take place.

It is easy to prove this experimentally by using the apparatus No. 1, Fig. 5, in which a bar can be made fast at one end and inclined, for instance, at an angle of 30 degrees below the horizontal. If we use No. 2, 20 inches long, the load suspended from the free end must be $4\frac{1}{4}$ pounds to produce buckling. We found at first in the vertical tests, and afterward in the modified angular tests, that $PL^2 = 1700$; hence we must have:

$$P_1 L^2 (\cos.^2 30° - \sin.^2 30°) = 1700$$

$$L^2 = 400;\ \cos.^2 30° = 0.75;\ \sin.^2 30° = 0.25.$$

Hence $$P_1 = \frac{1700}{400 \times 0.5} = 8.5 \text{ pounds.}$$

In reality, the weight required in the test is a little more than $8\frac{1}{2}$ pounds. This excess is due to the fact that, as soon as the load is applied, it causes a deflection of the bar, which augments the angle α. For thin wooden specimen the deflection is very great and must be taken into account. Thus, the load required is also slightly greater for the horizontal than for the angular and vertical tests.

We may write for any angle α the general formula:

$$P L^2 (\cos.^2 \alpha \pm \sin.^2 \alpha) = \frac{\pi^2}{4} E I, \tag{20}$$

to which the sign (+) must be employed when the component of P is a compressive force, and sign (—) when this component is tensile.

My first experiments were made upon beams supported at the ends and loaded at the center. In the absence of data concerning the question, it was thought that the beams buckled because they were long and thin—that is, *structurally weak*—but soon after the start it was observed that they obeyed a certain law. It was found also that the manner of applying the load had a great influence on the results. Thus, with a very thin beam, if the load is applied at the middle, but resting on the upper face, the system is very unstable, because the bending force, acting above the center of gravity of the beam, tends to upset this latter without actually buckling it. If the load is suspended by means of a wire terminated by a ring or a loop passing around the beam and resting on the upper face at the middle of the length, the center of gravity of the system is very low, and it would seem that the equilibrium is assured; but it is not so, and the beam is as unstable as in the first case, because the least oscillation of the load may cause an oblique pull on the upper edge, and thus upset the beam. After trying different methods for applying the load, one was found to give very regular results and was consequently adopted for all the experiments. It is the

same as the one used with the apparatus illustrated in Fig. 5, page 42. Two thin strips of wood, Fig. 12, were fastened at one end to the beam by a small bolt and nut, the bolt passing through a hole bored at the middle of the hight h on the neutral axis of the beam, and at the other end the load was suspended. The beam and load were thus solidly connected. In order to still further increase the stability of the system the ends of the beams were cut in such a way that, previous to the application of the load and barring the small deflection due to the weight of the beam itself, the points of support and the center of suspension of the load were situated on the same horizontal line, viz., the longitudinal axis of the beam. This excess of care naturally developed an increase of the elastic equilibrium of the specimen, but this was not especially noticed until the difficulties encountered in the angular tests exposed the true guiding action of the suspension strips. We have seen that when a beam supported at the ends buckled, the points of support and of suspension remained in the same vertical plane, but the beam assumed a double curvature in the form of an elongated S. This very misleading double curvature was caused entirely by

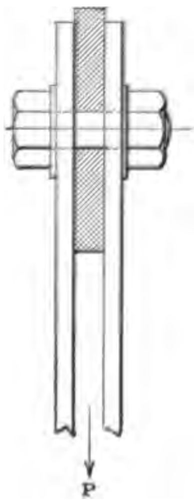
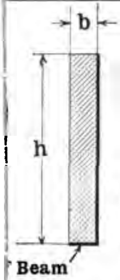


FIG. 12.



$E = 1380000$

$I = \frac{bh^3}{12}$ = moment of Inertia about neutral plane

$I_1 = \frac{b^3h}{12}$ = moment of Inertia about vertical axis of Section

$\frac{PL}{4} = f\frac{bh^2}{6}$ = bending moment — $f = \frac{3}{2}\frac{PL}{bh^2}$

$P_o = \frac{\pi^2 EI_1}{L^2}$

$h = 1\frac{1}{4}''$			$b = \frac{3}{16}''$		$h = 2''$		
$\frac{PL}{4}$	f	P_o	P	PL^2	$\frac{PL}{4}$	f	P_o
106.25	2147.4	8.42	19.	21964.	161.5	1292.	12.94
116.	2314.4	9.5	22.	22528.	176.	1408.	14.61
120.	2425.2	10.81	26.	23400.	195.	1560.	16.62
133.	2688.	12.41	29.	22736.	203.	1624.	19.09
139.75	2824.4	14.4	34.	22984.	221.	1768.	22.13
150.	3031.56	16.9	39.	22464.	234.	1872.	25.98
167.75	3390.3	20.1	47.	22748.	258.5	2068.	30.91
185.	3738.9	24.33	57.	22800.	285.	2280.	37.41
204.75	4138.1	30.03	70.	22680.	315.	2520.	46.18
230.	4648.4	36.01	88.	22528.	352.	2816.	58.45
245.6	4964.3	43.25					
253.75	5128.3	49.65					
				22683.2			

American Machinist

nent

PL

PL²
31700
31232
31050
29400
28054
27933
26862
29474.[illegible]

TABL

the suspension strips, which, forcing the center of the beam to remain in a vertical plane, effectively prevented it from twisting. Furthermore, the beam in developing one loop on each side of its axis increased the static stability of the system and caused an augmentation of the buckling board.

After the suspension parallelogram was devised and applied with such successful results to the beams held at one end and loaded at the other, the question clearly defined itself. The work done previously had been spent on a complex case; that is, a beam supported at the ends had supported at the middle of its length a load which guided it in a vertical plane. This case resembles somewhat that of a vertical column held at the base and sustaining at the top an axial load guided vertically.

It was then necessary to make anew the previous tests, this time employing the parallelogram in order to let the beam deviate or twist freely. The results were at once convincing, and the three laws were almost perfectly verified. The load observed for a beam of length L, was found to be that necessary to buckle the same beam supported at one end and loaded axially at the other, as per Euler's formula:

$$P = \frac{\pi^2 EI}{L^2}.$$

In Table 5, P is the load observed as applied with the strips, and P_0 is the theoretical load deduced from

the above formula, and with which that applied by means of the parallelogram almost perfectly coincided.

In Table 6 we have the results obtained with a built-up I-beam. The top and bottom flanges were glued to the web, and formed a solid combination. The load was attached to a strong wooden bar which had an end cut out so as to fit exactly the shape of the beam. This method was thought the best at the time, but afterward the parallelogram was tried, and the observed loads again coincided with the theoretical ones given in the P_0 column.

In Table 7 are given the results obtained with a built-up T-beam. The load was applied by means of a strong bar cut to fit the shape of the beam. But in the first position (⊥) the bottom flange was left free, the bar being fitted only to the vertical web. The different values observed in the two positions would seem at first to indicate that the beam can resist better in one position than in the other. But these differences are due to the guiding influence of the suspension bar, and also to friction at the supports. In the **T** position the load acts upon a flat horizontal surface and tends to keep it in the same plane, while the web is guided also vertically, consequently the resistance of the beam is somewhat higher than in the other position. This can be made clear by using the parallelogram, with which we would find the resistance alike for both positions were it not for the

friction at the supports, the loads coinciding with the theoretical P_0.

In these tables we see the gradual increase of the stress f. This stress does not seem to exert any influence upon the resistance of the solid, for the beam buckles only when the product of the load into the square of the length equals a certain constant.

A large number of tables could be added to those already cited, which would give the results of experiments performed on many specimens of various forms and on a few steel beams, but the writer, satisfied for himself that the laws advanced in this memoir have been verified, and certain that the present question will interest the mechanical world, would prefer to see the matter taken up by competent investigators, having at their disposal perfect testing apparatus, and who would confirm his own work or indicate necessary modifications of the formulas given here.

Since the theoretical laws of Euler seem to govern the question, *the experiments, in order to be conclusive, must conform absolutely to the requirements of the theory.* Thus, the limits of elasticity must not be trespassed, and the specimen must be held or supported and loaded *exactly* as the theory requires. It is proverbial that every rule has its exceptions, but in our case the rule or law, pure and simple, must be firmly established at first, and it will be easy afterwards to study the exceptions or

complex cases, one by one, as they are encountered in practice.

The first law of the elastic equilibrium of solids is very remarkable. It shows that a beam will buckle, whatever be its length, if the load applied is such that the relation $PL^2 =$ constant is satisfied; if $L = o$, $P = \infty$, and *vice versa*.

No less remarkable is the fact, proved by the angular tests, that the separate effects of one force unite into a common effort tending to destroy the equilibrium. From careful consideration of this fact results the finding of the key to many problems heretofore unsolved.

Since the components of one force, being in fact two distinct forces acting at the same point, combine their efforts, we may logically deduce therefrom that the efforts of two or more forces acting at various points on the beam may also combine towards the same object.

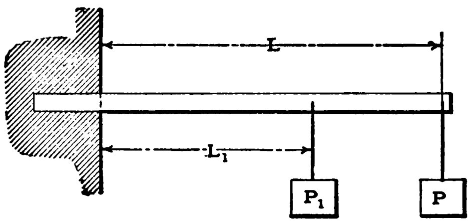


FIG. 13.

Thus, for a beam held at one end, loaded at the other and also at another point, Fig. 13, we may have the relation:

$$P L^2 + P_1 L_1^2 = \text{constant}. \qquad (21)$$

which is verified by very easily made experiments.

This being true for two forces is also true for any number of forces, and we have:

$$[p_1 l_1^2 + p_2 l_2^2 + p_3 l_3^2 + \ . \ . \ . \ + p_n l_n^2] = \text{constant}. \qquad (22)$$

For a beam uniformly loaded $p_1 = p_2 = p_3 = \ . \ . \ .$ $p_n = p =$ pressure per unit of length, whence

$$p\,(l_1^2 + l_2^2 + l_3^2 + \ . \ . \ . \ + l_n^2) = \text{constant}.$$

But the loads p_1, p_2, p_3, etc., being equally spaced if l is the distance between any two loads, we have:

$$p\,[l^2 + (2\,l)^2 + (3\,l)^2 + \ . \ . \ , \ + (n\,l)^2] = \text{constant}.$$

or

$$p\,l^2\,(1 + 2^2 + 3^2 + \ . \ . \ . \ + n^2) = \text{constant}.$$

The sum of the squares of the numbers from 1 to n, inclusive, is

$$\frac{n\,(n+1)\,(2\,n+1)}{1\,.\,2\,.\,3}.$$

But l is the unit of length and $n = n$ units of length $= L$, hence,

$$\frac{p\,L}{6}\,(L+1)\,(2\,L+1) = \text{constant} = \frac{\pi^2}{4}\,E\,I. \qquad (23)$$

This cannot well be proven experimentally, because it would be difficult to so suspend the loads as to insure to the beam complete freedom to deviate or buckle laterally.

But we will now analyze an experiment which completely proves the combination of the effects of forces.

A beam was tested in the manner shown by Fig. 14. It was supported at the ends and equally loaded at two points A and B equidistant from the center of the beam. These points were varied along the length, but always kept equidistant from the center, and the loads applied

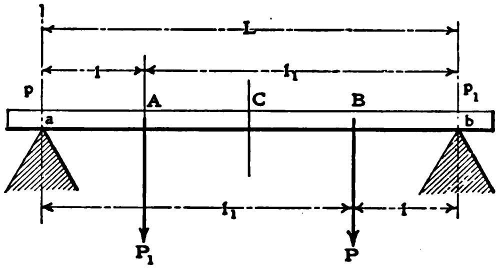


FIG. 14.

were always equal—that is $P = P_1$. The results are shown in Table 8. Here the relation $PL^2 =$ constant does not exist. Instead, we have the bending moment $PL =$ constant. This at first seems to indicate a radical modification of the first law, but we will see presently that it confirms it remarkably.

The question is much simplified if the effect of one load is studied independently of the other. Therefore, we may decompose the problem into two parts, viz.:

1. *A beam of length* L *supported at the ends sustains a load* P *applied at any distance* l *from one end;*

2. *The beam sustains two equal loads* P *and* P_1 *equidistant from the center.*

CASE I, FIG 15.

The reactions of the supports are p and p_1. The moments about B are $p\,l = p_1\,l_1$. We now have the well-known relations:

$$p + p_1 = P; \qquad l + l_1 = L;$$

$$p = \frac{P\,l_1}{L} = \frac{P\,(L-l)}{L}; \qquad p_1 = \frac{P\,l}{L}.$$

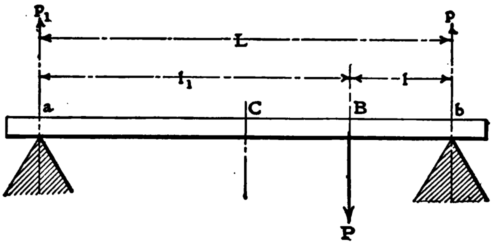


FIG. 15.

If the action of the force p was such as to cause the buckling of the part $B\,b$, we would have:

$$p\,l^2 = C = \frac{\pi^2}{4}\,E\,I; \; (C = \text{constant}).$$

But here to this action must be added that of force p_1. Thus we have $p\,l^2 + p_1\,l_1^2 = C$. Replacing p and p_1 by their values as above, then:

$$\left(\frac{P l_1}{L} \times l^2\right) + \left(\frac{P l}{L} \times l_1^2\right) = C,$$

or

$$\frac{P}{L}(l_1 l^2 + l l_1^2) = C;$$

and finally after reduction:

$$P l l_1 = P(L - l) l = C. \qquad (24)$$

This can be demonstrated in another manner as follows: The actual moment at the middle of the beam due to force p is

$$p_2 \times \frac{L}{2} = p l.$$

In other words, the force p_2 is that which, acting at b, would produce about C a moment $p_2 . \frac{L}{2}$, equivalent to the actual moment $p l$. We have then $p_2 = 2 p \frac{l}{L}$.

Similarly for p_1 the corresponding force p_3 would be

$$p_3 = \frac{2 p_1 l_1}{L}.$$

The buckling of one-half of the beam would be due to the combined efforts of p_2 and p_3 acting upon a length $\frac{L}{2}$, therefore:

$$p_2\left(\frac{L}{2}\right)^2 + p_3\left(\frac{L}{2}\right)^2 = C,$$

or

$$\frac{2 p\, l}{L} \cdot \frac{L^2}{4} + \frac{2 p_1 l_1}{L} \cdot \frac{L^2}{4} = C.$$

Replacing p and p_1 by their values as before:

$$\left(\frac{2\, P\, l\, l^1}{L^2}\right)\frac{L^2}{4} + \left(\frac{2\, P\, l\, l_1}{L^2}\right)\frac{L^2}{4} = C.$$

Finally we have again

$$P\, l\, l_1 = C = \frac{\pi^2}{4} E\, I.$$

If the load is applied at the middle of the beam,

$$l = l_1 = \frac{L}{2}, \text{ therefore } P\, l\, l^1 = \frac{P\, L^2}{4} = C;$$

whence $$P\, L^2 = 4\, C = \pi^2 E\, I,$$

as found by the experiment.

CASE II. FIG. 14.

The reactions of the supports are:

at a,	at b,	
p	p_1	. . . due to P.
p_1	p	. . . due to P_1.

We have here also the known relations:

$$p + p_1 = P = P_1, \qquad l + l_1 = L,$$

$$p = \frac{P\, l_1}{L} = P\left(\frac{L - l}{L}\right), \quad p_1 = \frac{P\, l}{L}.$$

The bending moment about A or B: $(p + p_1)\,l = Pl$ is maximum and constant from A to B. For instance, at C the moment is

$$2\left(p_1 \cdot \frac{L}{2}\right) = Pl.$$

The sum of the effects of the two forces P and P_1, causing the buckling of the beam, we must find as before:

$$2\left(\frac{2Pl}{L} \cdot \frac{L^2}{4}\right) = PLl = C = \frac{\pi^2}{4} EI.$$

$\left(\frac{2Pl}{L}\right)$ is the value of the force which would produce upon a length $\left(\frac{L}{2}\right)$ an effect equivalent to that due to the force P acting on a length l.

If we call P_0 the total load carried by the beam, then $P + P_1 = P_0$, and therefore

$$PLl = \frac{P_0}{2} Ll = C.$$

When this load is applied at the center of the beam, $l = \frac{L}{2}$, whence

$$\frac{P_0}{2} Ll = \frac{P_0 L^2}{4} = C. \qquad (25)$$

and we have as before,

$$P_0 L^2 = 4\,C = \pi^2 EI.$$

Thus we see that equation (25) explains the results obtained in the experiments. The bending moment was found to be constant, the equation shows, in effect, that $PL = C =$ constant.

COROLLARY, FIG. 16.

The beam sustains a load P_0 *uniformly distributed.*

In Case II. we have considered one system of forces equidistant from the center of the beam but, for a beam uniformly loaded, the buckling is evidently due to the

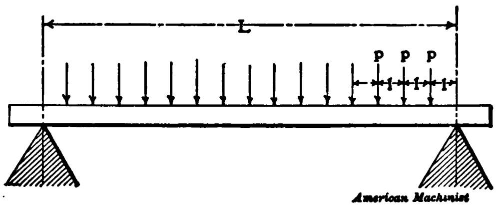


FIG. 16.

sum of the efforts of a great number of such systems. Consequently, if p is the load per unit of length, we may assume that the beam sustains a number of forces distant from one another a length l equal to the unit of length, Since two symmetrically applied forces, such as p_0, constitute a system, we must have:

$$\Sigma p_0 L I = C; \tag{26}$$

or $$p_0 L [l + 2l + 3l + \ldots + nl] = C.$$

Hence $$p_0 L I \left(\frac{n(n+1)}{2}\right) = C.$$

But $p_0 = p$, $l = 1$, $n l = \frac{L}{2}$; therefore, the limit of the elastic equilibrium of a beam supported at the ends and uniformly loaded may be expressed by the equation:

$$p L^2 \left(\frac{L+2}{2}\right) = 4 C = \pi^2 E I;$$

or, since $p L = P_0 =$ total load applied on the beam,

$$P_0 L \left(\frac{L+2}{2}\right) = \pi^2 E I. \tag{27}$$

A great number of cases in which forces tend to destroy the elastic equilibrium of beams held or supported in various manners remain to be treated, but for the solution of our *simple problem* we must content ourselves with the few examples briefly analyzed in the present memoir. However, the writer is preparing a voluminous work in which the principal questions likely to be encountered in practice are given proper treatment.

Because the few tables given in this memoir concern only beams of small section, the largest being 2 x ¼ inches, many readers may think that the whole proceeding is more of the nature of play than of serious study, and that the conditions dealt with are not those likely to be encountered in practice.

It should be remembered that from the start the

object of the investigation was the discovery of the principle governing the case. The writer is confident that this object has been attained. By means of very simple apparatus, with specimen of diminutive sizes—permitting the selection of almost perfect material—requiring relatively small loads to cause distortion, many objectionable features were eliminated from the tests. For instance, the friction at the points of support of the beams was not of such importance as would have been the case with heavy loads causing the supports to indent the beams, and thus guide these in a certain direction, preventing them from twisting naturally (see experiments on T-beams); the smallness of the specimens and of the loads afforded easy handling of the objects and also—which is most important—a more complete observation of the phenomena taking place. Not in defence of the method employed by the writer, but simply to strengthen the confidence of the readers and of those who would like to verify the results given here, it may be said that the Science of the Resistance of Materials owes more to laboratory experiments on small scale than to those made on large specimen. Besides, although the beams employed were rather diminutive, it suffices to glance at the values of f given in the tables to be convinced that in many cases the breaking strength of the specimen was almost attained. (In several occasions $f =$ nearly 5,000 pounds per square inch.)

Should subsequent tests undertaken by skillful experimenters upon specimen of various forms and composition prove the necessity of modifying by suitable coefficients the formulas advanced here, the fact remains patent that, in principle, there exist three principal laws governing the elastic equilibrium of solids. These laws, unknown until now, may be expressed thus:

FIRST LAW.—The load causing the buckling of a beam through flexure is inversely proportional to the square of the length upon which it acts directly.

SECOND LAW.—The load is directly proportional to the cube of the thickness and to the depth of the solid or, generally, directly proportional to the moment of inertia of the section of the beam considered, taken in reference to an axis passing through the center of gravity of the section, parallelly to the direction of the load.

THIRD LAW.—The total effect of one or several forces or of their components is equal to the algebraic sum of their individual effects.

THAT SIMPLE PROBLEM.

Stepping now from the laboratory into the study, we find on the table the dear old question patiently awaiting our good pleasure. After a careful dusting let us read it again:

It is required to design a beam having a minimum volume and which, firmly held at one end, must sustain at the other a load P *acting perpendicularly to the length* L.

Although we have accomplished much, much remains to be done, and it behooves us at present to make an inventory of our resources.

Our text-books gave us for any bending moment the equation:

$$P x = f \frac{I}{a}. \qquad (1)$$

Whence the value of the modulus of resistance is derived in a general way, the terms P, x and f being known, thus:

$$\frac{I}{a} = \frac{P x}{f}.$$

This equation is true for any form of section, but absolutely indeterminate for all but plain sections such as the circular and the square; for the rectangular, having $b\,h$ for dimensions, it is necessary to assume a value for one of these in order to obtain the other. Therefore any value of b, for instance, will satisfy the equation, but, as we have previously seen, the area of the section and consequently the volume of the beam varying directly as the width, it seems advantageous to make this latter very small. Then our books and our experience warn us not to make the beam too thin for fear of lateral collapse.

We have found the consideration of the deflection of of the beam a most valuable auxiliary, because by specifying the degree of rigidity necessary for a given case, we find by equation (7):

$$h = \frac{2}{3} \cdot \frac{1}{m} \cdot \frac{f}{E} \cdot \frac{L}{n}$$

which expresses the hight of the section, irrespective of the shape of the latter. For a rectangular section this value introduced in equation (1) determines b. But, although the dimensions of the sections are now known, the stability of the solid is by no means assured, and the latter may collapse before the stress f developed in the beam has attained the value originally assigned to it.

From our experiments was obtained the expression:

$$P L^2 = \frac{\pi^2}{4} E I_1,$$

which asserts positively the limit at which a beam of length L with a section having a moment of inertia I_1—taken with reference to an axis perpendicular to the neutral plane and passing through the center of gravity of the section—will collapse when supporting a bending load P.

Before proceeding further, let it be understood that this is the absolute limit and that, like the breaking strength of the solid, it must not be reached in practice. Consequently a factor of safety must be selected. During

the tests it was observed that the specimen could be vigorously vibrated until the load applied reached $\frac{3}{4}$ of the limit load, after which care was necessary until $\frac{7}{8}$ of the limit was reached, at which point the specimen became very lazy and vibrating had to be checked, because any vibration had a tendency to assume an exaggerated amplitude, although actual collapse took place only with the limit load.

The writer considers a factor of safety of 2 amply sufficient in all cases in which the beam is held and the load is applied in the proper way, as explained before. There will, undoubtedly, be quite a variation of opinion regarding this value. It would be wrong to adopt definitely one factor for all cases, irrespectively of the manner in which the load is applied, and, likewise for f, discernment will have to be exercised.

It is also worthy of remark that when the beam is abruptly vibrated the point of application of the load may be displaced, especially if the load is not rigidly fastened to the beam. In such a case the aspect of the question is changed and the solid becomes subjected to combined stresses, requiring consequently a special treatment differing from that followed here.

Quite a number of beams so shaped as to be very nearly of uniform strength were tested and found to be weaker than those having a uniform section; but it has

not been possible so far to establish a well-defined rule governing the case. By employing formula:

$$P L^2 = \frac{\pi^2}{4} E I$$

modified by a factor of safety of 2 no fear that collapse will occur need be entertained.

Finally, we may say that everything considered it may be found convenient to adopt for the collapse limit the same factor of safety used for the breaking strength of the beam, thereby greatly simplifying the matter.

THE BEST FORM OF SECTION.

Now, if a rectangular section calculated for strength and rigidity is found to be weak for resisting collapse, it must be altered. By increasing b the required inertia moment I_1 may be attained, but the volume of the beam will be proportionally increased. It is true that the strength will be augmented, but no benefit will be derived from the fact, since the originally adopted value of f insured safety. By modifying the shape of the section the beam can be made to satisfy the three conditions of strength, rigidity and stability, and at the same time its volume may be kept close to the minimum limit.

The most convenient and widely employed is evidently the I-shape. The question is now to determine the proportions of the different parts of the section, or first, what must the area be? As generally treated in practice, the central web is calculated to resist the transverse shearing, although it combines also with the top and bottom flanges to resist flexure. Neglecting for the present this central web, let us assume that the section is composed only of two equal and parallel rectangles, Fig. 17. Its principal moments of inertia are:

$$I = \frac{B}{12}(H^3 - h^3)$$

$$I_1 = \frac{B^3}{12}(H - h).$$

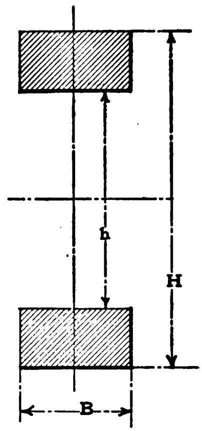


FIG. 17. I-BEAM WITHOUT WEB.

The area $A = B(H - h)$ will be a minimum with $h = H$ when $A = o$; in other words, if we make the thickness of the rectangle infinitely small, B will become infinitely great and, at the limit, the top and bottom parts will be only straight lines of infinite length. It seems therefore advantageous to reduce the thickness. A very simple experiment will show that great care must be exercised in this matter. If a prism of rubber a few inches long, and, say, $\frac{1}{2} \times \frac{3}{4}$ inch is carved longitudinally

so that its section be as shown in full lines in Fig. 18, we will have an I-beam, very flexible it is true, but this is a valuable quality for our need. Let us bend it in the form of an arc of circle; it will then be deformed as shown in dotted lines. We see, first, that the top and bottom flanges have not remained parallel in the section;

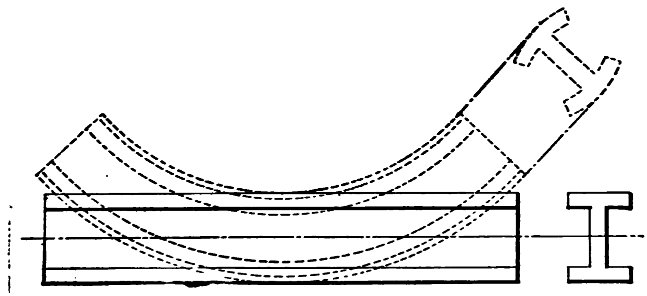


FIG. 18. ACTION OF A FLEXIBLE I-BEAM.

second, that their middle has been kept in place by the rigidity of the central web; third, that their edges, being unsupported, have been brought nearer to the neutral axis, and—as the tensile or compressive stresses are proportional to the distance of the fibers from the neutral axis—they are consequently less strained than the middle portions. Therefrom we conclude that, because of this deformation, the beam has been weakened. The same thing would occur to a beam made of rigid material, like iron, but of course the deformations would not be so pronounced in actual practice. However, for

the purpose of testing, if the specimen was bent excessively—at right angle, for instance—the deformation would be similar to that of the rubber prism.

To the writer's knowledge, this very important point has never been treated analytically in such a manner as to establish a practical solution, and our authorities generally advise us benevolently "not to make the flanges of the beam too wide and too thin for fear of collapse. Right here is a wide field for investigation, as well mathematical as practical. The problem is indeed very attractive, etc. . . ." Thus left in a lurch, having nothing else to do, if perchance we cast a glance on the illustrated lists of commercial rolled iron and steel beams, such as used on bridge and building construction, we notice that the sections seem to be made with a view to economy as well as strength; the material seems to be distributed in the most effective manner, and the flanges are tapered, apparently to guard against collapse. Although rather lean, the section looks very strong and rigid. Evidently the proportions have not been chosen at random; on the contrary, they must certainly have been judiciously determined from carefully made experiments. Since these shapes have been sanctioned by general practice, it would be wise to take them for models, or at least to use them as standards of comparison, and especially so in the absence of any other reliable data.

The moment of inertia of an I-beam has for its expression:

$$I = \frac{BH^3 - bh^3}{12}.$$

We have here one known quantity (H)—determined by the deflection formula—and three unknown: (B), (b), (h). This equation is consequently indeterminate, and values must be assumed for some of these unknown in order to obtain a solution. The same remark applies to the expression for the area of the section:

$$A = BH - bh.$$

These expressions are much more complicated in the case of the commercial rolled beam sections. Hence it does not seem possible to make comparison analytically between one of the latter and a plain section. However, the writer has worked out a formula expressing very closely the areas of most of the rolled I-beams made in the United States as well as in Europe. It is derived from the comparison—almost unwarranted, but the end justifies the means—of a *hollow square box beam* with a rolled I-beam having the same hight of section, same area A and same moment of inertia I. It is well known, of course, that the moment of inertia of a hollow box section is the same as that of an I section having the same hight, width and thickness of top and bottom flanges, the sum of the thicknesses of the lateral webs

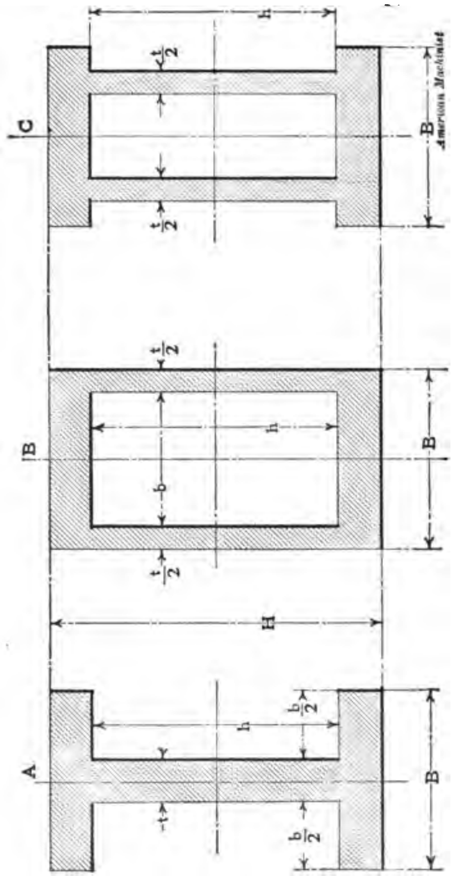


FIG. 19. I AND HOLLOW BOX BEAMS.

being equal to the thickness of the central web, as in Fig. 19. Knowing I and H, the question is to find A.

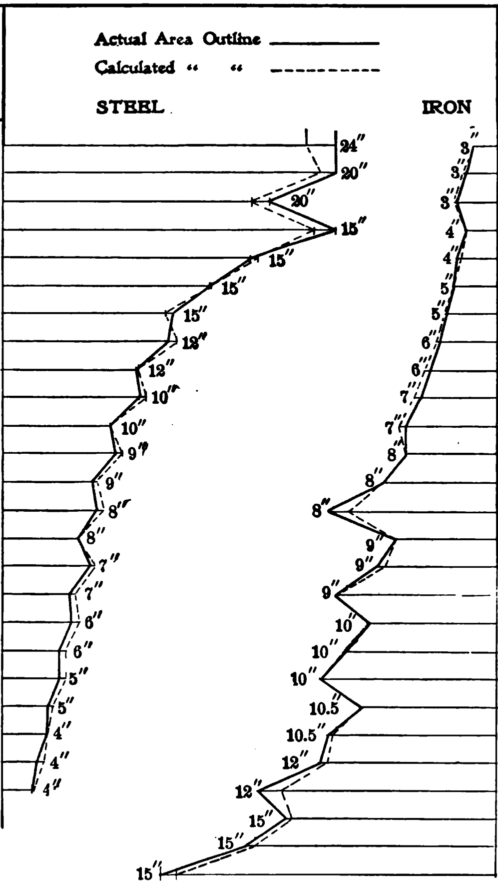
For a hollow square section:

$$I = \frac{H^4 - h_1^4}{12}$$

$$A = H^2 - h_1^2;$$

Steel

Actual Area	Calculated Area
23.5	21.5
23.5	21.4
18.8	17.6
23.5	22.2
17.6	18.
14.7	14.8
12.	11.5
11.7	12.4
9.4	9.5
9.7	10.2
7.5	7.5
7.9	8.5
6.2	6.5
6.5	7.
5.3	5.2
5.9	6.5
4.6	5.
4.7	5.3
3.8	4.2
3.8	4.3
3.	3.3
2.9	3.
2.2	2.7
1.8	2.



Iron

Actual Area	Calculated Area
1.7	1.5
2.2	2.5
2.9	3.2
2.1	2.2
2.9	2.6
3.	3.2
3.6	3.8
4.1	4.3
4.8	5.2
5.4	5.9
6.6	7.
6.5	6.5
8.1	8.1
12.2	10.5
7.1	7.2
8.6	8.
11.6	11.5
9.	9.1
10.8	10.9
12.6	12.5
9.5	9.75
12.	11.75
12.6	12.
17.	15.3
15.	14.5
18.	17.3
24.	22.8

FIG. 20. ACTUAL AND CALCULATED AREA OF CARNEGIE I-BEAMS.

from these two equations we obtain:

$$h_1^2 = \sqrt{H^4 - 12 I} = H^2 - A.$$

Whence

$$A = H^2 - \sqrt{H^4 - 12 I}. \tag{28}$$

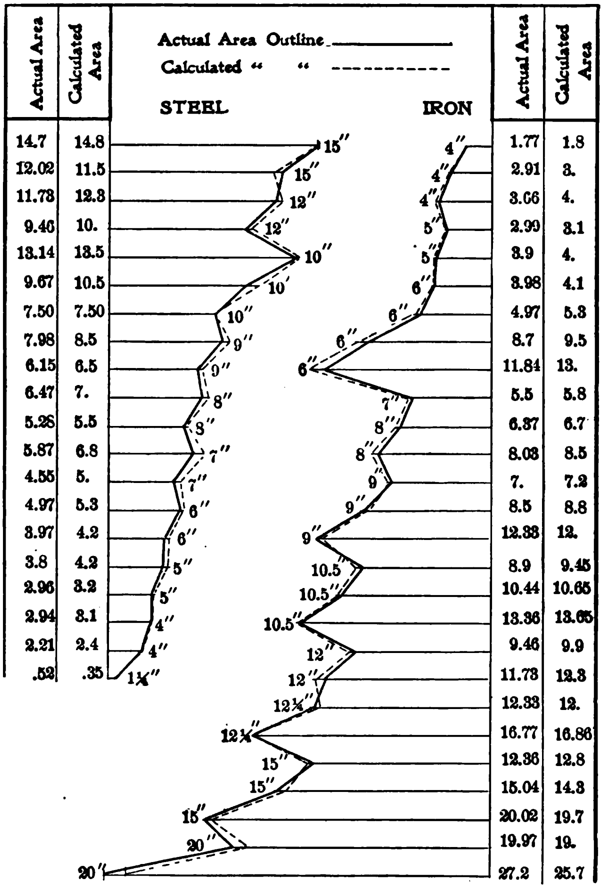


FIG. 21. ACTUAL AND CALCULATED AREA OF NEW JERSEY STEEL AND IRON CO.'S I-BEAMS.

If we apply this formula to the Carnegie I-beams—steel and iron—the properties of which are given in the well-known pocket-book of shapes, and also to those of

the New Jersey Steel and Iron Company, we obtain very conclusive results. These latter can be shown to better advantage in the form of a diagram, as in Figs. 20 and 21. The abscissæ are simply equal distances for spacing the shapes and have no other meaning, but the ordinates are drawn to scale and indicate the areas of the sections, The top of the ordinates are joined by straight lines. The full lines indicate the areas given in the books of shapes, and the dotted lines those derived from formula (28). It is thus seen that the two series of values are nearly identical.

When such a simple formula—even if found entirely by chance—agrees so closely with good practice, it is advisable, because of the lack of reliable data, to give it conscientious trial so as to find if it is necessary to modify it before final adoption.

For our case we will consider that equation (28) expresses the minimum (practical) area of the greatest section of the beam.

THE CENTRAL WEB.

Text books do not contain precise formulas by which the web may be calculated; they are unanimous, however, in stating that the thickness must not be too small for fear of collapse. We have solved experimentally the

question of the lateral collapse of the beam and have excavated the sectional area from good experience sanctioned by practice. Let us now draw on our imagination for a reasonable hypothesis of the real function of the web.

To begin with, let us assume that the beam is trussed as shown in Fig. 22. The parts *a*, *b*, *c* are in tension, *d*, *e* in compression. They are respectively the top and

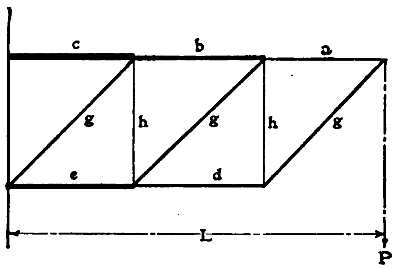


FIG. 22. TRUSSED BEAM.

bottom flanges of the beam. Coincidently with the stresses developed in them they increase in size as they recede from the point of application of the load; thus the section of *c* is greater than that of *b*, which in turn is greater than that of *a*. The diagonals *g* are in compression and are equally strained. The bars *h* resist equal tensive stresses. These bars and diagonals are to the beam illustrated here what the central web is to a solid beam. The diagonals must be treated as struts and the least moment of inertia of their transverse section must be large enough to insure their safety against

buckling. If, out of the total amount of the material composing bars and diagonals, we form a thin plate filling the space between the top and bottom flanges, the result is a solid beam. It would seem at a first glance that no important change has been effected, but with a closer attention we discover that a great complication of stresses has been established in the web. The compressive forces—shown in diagonal lines in Fig. 23—are still there, as are also tensile forces (vertical lines);

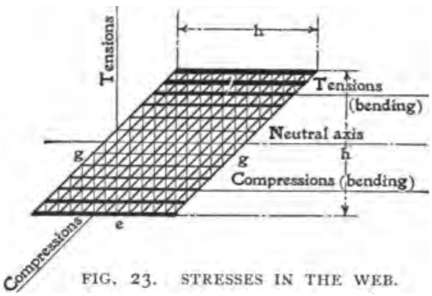


FIG. 23. STRESSES IN THE WEB.

in addition to these, the top and bottom flanges, being now rigidly fast to the web, transmit to it bending stresses (horizontal lines) proportional to the distance of the fibers from the neutral axis. The determination of the resultant effects of all these stresses at work on a transverse section of the beam—the section being assumed plane before the application of the load—belongs to the domain of mathematical analysis of the highest order, on which, for obvious reasons, we will not attempt to trespass.

But although all these stresses combine towards a common effect, their individuality is not obliterated and, for the safety of the structure, each kind of stress must be counteracted. Consequently, since our experiments have taught us that prisms are liable to buckle when under compression produced directly or resulting from bending, and because we may with reasonable certitude believe that the central web of a beam is subjected to similar compressive strains that obtain in the diagonals of a latticed girder, it seems proper to employ, for determining the thickness of the web, the rules governing the collapse of struts.

By adopting 45 degrees for the angle of inclination of the diagonal we are in accord with good practice. Thus we are to treat as a strut a panel of thickness t, length $= \frac{h}{\cos. 45}$, width $= h \times \cos. 45$, loaded axially and parallelly to the diagonal with a weight $= \frac{P}{\cos. 45}$. If we consider that the panel is not guided at the top and bottom—as that is most likely the case—the formula applicable here is

$$P_0 L^2 = \pi^2 E I_0.$$

But for $P_0 L^2$ we must write:

$$\frac{P}{\cos. 45} \cdot \frac{h^2}{\cos.^2 45} = \frac{P h^2}{\cos.^3 45}.$$

Using 2 as a factor of safety, we have:

$$\frac{P h^2}{\cos.^2 45} = \frac{\pi^2 E \, . \, t^3 h \cos. 45}{2 \times 12}$$

Solving for t, we have:

$$t = \sqrt[3]{\frac{96 \, P h}{\pi^2 E}}. \tag{29}$$

Our hypothesis gives us a very simple formula. While it cannot be claimed that the reasoning leading to this result is irrefutable in some parts—leaving aside the free assumption of the inclination angle of the diagonal—where it is primarily at fault is in dealing too summarily with such an important subject.

THE WIDTH OF THE SECTION.

If we decompose the transverse section of an I-beam into several elements, as shown in Fig. 24, we do not change its moments of inertia, for we have:

$$I = \frac{B H^3 - b h^3}{12} = \frac{t H^3 + 2\left(\frac{b}{2}\right)(H^3 - h^3)}{12}$$

in other words, the total moment of inertia is equal to the sum of the moments of inertia in the parts, taken with respect to the same neutral axis.

The unknown quantity in this case is B.

The area of this section is

$$A = th + 2\left(\frac{b}{2}\right)(H - h).$$

Calling I_2 the moment of inertia of the flanges alone and A their area, we have:

$$I_2 = \frac{12 I - t H^3}{12} = \frac{b(H^3 - h^3)}{12};$$

$$A_2 = A - th = b(H - h); \qquad (30)$$

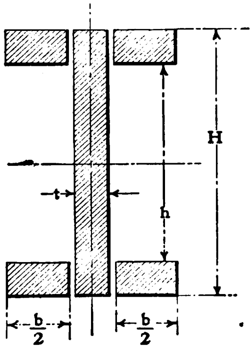


FIG. 24. ELEMENTS OF AN I-BEAM.

whence, by combining:

$$\frac{12 I_2}{A_2} = \frac{H^3 - h^3}{H - h} = H^2 + Hh + h^2.$$

Solving for h, we have:

$$h = -\frac{H}{2} \pm \sqrt{\frac{12 I_2}{A_2} - \frac{3H^2}{4}}$$

$$= -\frac{H}{2} \pm \sqrt{\frac{12 I - tH^3}{A - th} - \frac{3H^2}{4}}. \qquad (31)$$

This value introduced into equation (30) gives us b, and as $b + t = B$ the section is entirely determined as far as strength and rigidity are concerned. It remains to calculate the moment of inertia I_1 and compare it with that required to insure safety against lateral collapse. If this value is superior or equal to the required one, the problem is solved; if it is only slightly inferior to it, the flanges may be widened a little but kept of the same thickness, the increase of area resulting therefrom being

negligible; if it is greatly inferior, two ways may be used to remedy the trouble:

(I) Calculate the flanges anew for the required inertia moment;

(II) Divide the central web in two parts and make the section as shown in Fig. 19, shapes B or C.

EXAMPLE.

To illustrate the use of the formulas established, let us undertake to solve the following problem:

To determine the maximum transverse section of a beam of uniform strength held horizontally at one end and loaded vertically at the other. The weight of the beam is not to be taken into consideration and the following data are furnished:

Length of beam = 100 inches = L,

Load applied =40,000 = P.

Modulus of elasticity = 28,000,000 = E.

Allowable fiber stress = 15,000 pounds per square inch = f.

Allowable deflection = $\frac{L}{400} = \frac{L}{n}$.

Deflection coefficient depending on shape of beam $= \frac{2}{3} = m$.

SOLUTION.

Hight of section $= H$.

By formula (7)

$$H = \frac{2}{3} \cdot \frac{1}{m} \cdot \frac{f}{E} \cdot \frac{L}{n}.$$

Hence

$$H = \frac{2}{3} \cdot \frac{3}{2} \cdot \frac{15,000}{28,000,000} \cdot 100 \times 400 = 21.428'' :$$

say $$H = 21\tfrac{1}{2}''.$$

Bending moment,

$$P\,L = f\frac{I}{a} \qquad a = \frac{H}{2}.$$

Moment of inertia,

$$I = \frac{H\,P\,L}{2f}.$$

Hence

$$I = \frac{21.5 \times 40,000 \times 100}{2 \times 15,000} = 2,866.66.$$

Moment of resistance,

$$\frac{I}{a} = \frac{2\,I}{H} = \frac{2 \times 2,866.66}{21.5} = 266.66,$$

If the section is made rectangular

$$\frac{I}{a} = \frac{B\,H^2}{6}.$$

Whence

$$B = \frac{6\,I}{a\,H^2} = 3.4613''.$$

Area of section,

$$A = BH = 3.461.3 \times 21.5 = 74.418 \text{ square inches.}$$

In the case of an I section the area, as per formula (28), would be

$$A = H^2 - \sqrt{H^4 - 12\,I}.$$

Hence

$$A = (21.5)^2 - \sqrt{(21.5)^4 - 12 \times 2866.66} = 38.85 \text{ sq. inches.}$$

By comparing these two values we see how advantageous the I section is, its area being about one-half that of the rectangular section. If we adopt the former, it remains to determine the thickness of the web and the sizes of the flanges, for the hight of section is the same in both cases.

Thickness of the web, as per formula (29).

$$t = \sqrt[3]{\frac{96\,P\,H}{\pi^2\,E}} = \sqrt[3]{\frac{96 \times 40,000 \times 21.5}{\pi^2 \times 28,000,000}}$$

$$t = 0.6685''.$$

The distance h between flanges, Fig. 25, is given by equation (31):

$$h = -\frac{H}{2} \pm \sqrt{\frac{12\,I - t\,H^3}{A - t\,H} - \frac{3\,H^2}{4}}.$$

Hence

$$h = -\frac{21.5}{2} + \sqrt{\frac{(12 \times 2,866.66) - (0.6685 \times 21.5^3)}{38.85 - (0.6685 \times 21.5)} - \frac{3 \times 21.5^2}{4}}.$$

$$h = 17.31''.$$

From equation (30) we obtain:

$$b = \frac{A - th}{H - h} = \frac{38.85 - (.6685 \times 17.31)}{21.5 - 17.31} = 5.818''.$$

And finally,

$$B = b + t = 5.818 + 0.6685 = 6.51''.$$

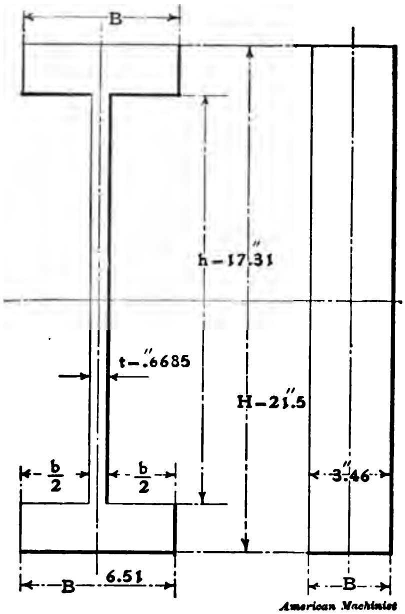


FIG. 25. A PRACTICAL EXAMPLE.

The section is now completely determined (see Fig. 25) and the problem is solved. However, the sizes obtained through the formulas—such as the thickness of the web, for instance—may have to be modified to facilitate the

process of manufacture. Fillets may be provided at the junctions of the web and the flanges; the angles may be rounded, etc.; but we have found in a simple manner, and without hesitation, the safe and economical limits which are of all importance in this case.

If any one, thoughtlessly, were to make the section rectangular and of the same area as that found for the I shape, he would have cause for regret, as we will presently see.

By introducing the value of $A = BH$ into the equation of the bending moment $PL = f\dfrac{BH^2}{6}$,

we have
$$PL = Af\frac{H}{6},$$

whence
$$H = \frac{6PL}{Af} = \frac{6 \times 40,000 \times 100}{38.85 \times 15,000} = 41.185''$$

$$B = \frac{A}{H} = \frac{38.85}{41.185} = 0.943.$$

The moment of inertia I_1 taken with respect to the vertical axis of the section is:

$$I_1 = \frac{B^3 H}{12} = \frac{(0.943)^3 \times 41.185}{12} = 2.881.$$

The moment of inertia I_1^1 required to insure safety against lateral collapse may be derived from equation:

$$PL^2 = \frac{\pi^2}{4} EI_1^1.$$

Employing a factor of safety of 2, we have:

$$I_1^1 = \frac{8\,P\,L_2}{\pi^2 E} = \frac{8 \times 40{,}000 \times 100 \times 100}{\pi^2 \times 28{,}000{,}000} = 10.58.$$

Comparing the values of I_1^1 and I_1 we see that the latter is nearly four times too small and that the beam, if made with such a rectangular section, would immediately collapse when the load is applied.

But as originally designed, with a hight $H = 21.5''$ and base $B = 3.46''$, the moment of inertia I_1 would be:

$$I_1 = \frac{(3.46)^3 \times 21.5}{12} = 74.29,$$

and the beam would have ample strength to resist lateral collapse.

As for the I section, its moment of inertia I_1 is:

$$I_1 = \frac{(B^3 - t^3)\,(H - h) + H t^3}{12}.$$

Hence

$$I_1 = \frac{(\overline{6.51^3} - \overline{.6685^3})\,(21.5 - 17.31) + (21.5 \times \overline{.6685^3})}{12}$$

$$I_1 = 95.706.$$

This value insures perfect security against lateral collapse.

THE SHAPE OF THE BEAM.

For a beam of length L rigidly held at one end and supporting at the other a load P producing flexure only, the bending moment Px for any section distant a length x from the point of application of the load is equal to the moment of resistance $f\frac{I}{a}$ of the section. As we have seen previously, this is expressed by equation (1): $Px = f\frac{I}{a}$ in which x is a quantity varying from zero at the loaded point to L at the holding point. P being constant, it follows that the second term must vary in order to preserve the equation. If we make f a constant, the modulus of section $\frac{I}{a}$ will vary directly as x and the beam in such a case is called a solid uniform resistance. If $\frac{I}{a}$ is made constant, the section of the beam is uniform throughout the length. The area A of the transverse section fluctuates in the same manner as $\frac{I}{a}$ and is therefore greatest for the value of this latter corresponding to $x = L$. For $x = o$ we have $\frac{I}{a} = 0$ and the area $A = o$. But as the load must have a support, the end of the beam,

instead of having a sectional area $= o$, is always made substantial enough to resist the vertical shearing produced by the load. Apart from this consideration we can see that a beam has a lesser volume when made of uniform strength instead of with a uniform section.

The preceding remarks are well known and are extensively developed in text-books. The quantity $\frac{I}{a}$ is indeterminate in this sense that it applies to any form of section, regular or otherwise, and consequently it shows that a beam can be made uniformly strong in an infinity of manners. It does not follow that the volume will remain constant whatever is the shape selected for the beam; on the contrary, variations in shape generally cause variations of volume. Since this latter may vary so widely there must be a certain shape for which the volume of the beam is a minimum. The truth of this assertion is well established and for plain transverse sectioned beams the shape having the lesser volume can be mathematically determined—and with ease. For instance, in Reuleaux's Constructor there are shown fifteen forms of beams of uniform resistance to flexure, together with the equations of the shape and of the volume of each solid. The largest or main section being calculated and the shape of the beam selected, this latter may be delineated according to the proper equation, thus saving the trouble to calculate a great number of

sections by a cut and try method.* The fifteen forms given by Reuleaux are meant only for beams of rectangular section, but we will see later on that they can be utilized for any form of section, under certain conditions. Among them, that which has a minimum volume, has a section with a constant hight H and a varying width B.

It is not so easy to proceed when the form of the section is a complicated one—like I or T, for instance. Each section to be calculated must generally be roughly estimated at first, and finally determined by successive approximations, The work involved by this process is, of course, very tedious, and the results obtained thereby have not all the exactness that could be desired.

In the following the writer gives a new law which which will be found of great help to the delineator, in that it will enable him to determine rapidly and rigorously the form of any beam of uniform strength.

THE NEW LAW.

The maximum transverse section of a beam of length L *being divided* (*rightly conceived as such for the purpose of delineating only, the adherence of the parts being unaffected*) *into any number of parts—equal, similar, or otherwise :*

1. *Any one part may be treated independently of the others, provided that the same form of treatment is applied to the others one by one or altogether;*

2. *Each part may be eonsidered as the transverse section of a beam or elementary prism of length* L;

3. *The size of the transverse section of sueh a prism at any point along* L *depends only on the value of the bending moment* P x *at that point;*

4. *Consequently each prism will be of uniform strength, and the combination of all the prisms or the solid beam will also be of uniform strength.*

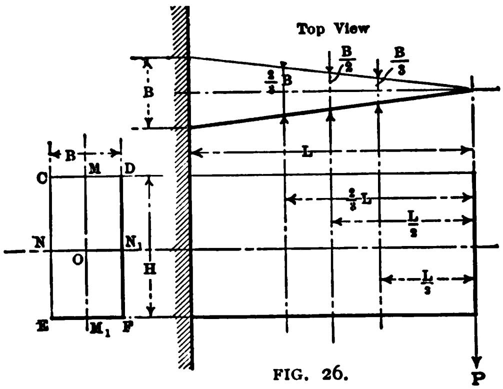


FIG. 26.

DEMONSTRATION.

Let Fig. 26 represent a beam held at one end and loaded at the other. For simplicity's sake the transverse section is made rectangular and the equation of the bending moment is, as we have seen before,

$$P x = f \frac{B H^2}{6}.$$

If we make the hight H constant, the stress f being constant—since the beam must be of uniform strength—we have for the value of B at any section distant a length x from the point of application of the load:

$$B = K x, \text{ where } K = \text{constant} = \frac{6 P}{f H^2}.$$

H must be determined at first for the maximum section where $x = L$, then we see that B varies directly as x, and, since $B = K x$, is the equation of a straight line, the plan or top view of the beam will be, as shown, a rectilinear triangle. If B is the width of the main section for $x = L$ then for $x = \frac{L}{3}, \frac{L}{2}, \frac{2 L}{3}$, etc., the width of the sections will be respectively $\frac{B}{3}, \frac{B}{2}, \frac{2 B}{3}$, etc.

Now let the rectangle $C D E F$ represent the main section, $N N_1$ is the neutral axis and $M M_1$ the vertical axis. We may conceive this section as being the sum of the two rectangles $C M M_1 E$ and $M D F M_1$ equal to one another, each of which is the section of a prism of hight H, base B_1 and length L, sustaining at one end a load $\frac{P}{2}$ or half the total load. Keeping the hight H constant, the width B_1 of each prism for any length x will be

$$B_1 = \frac{K x}{2},$$

the sum of the widths will be

$$2 B_1 = 2 \left(\frac{K x}{2}\right) = B,$$

or just what we had previously for a whole prism of the same length.

It is evident that the section being divided into any number—say, an infinity—of rectangles of same hight H, if all are treated similarly, the sum of the widths of the prisms will be equal to that B of the solid prism for any length x.

We can prove in the same manner that if the section is divided into any number of parts the constant hight H will equal the sum of the individual hights of these parts, and the width B will likewise equal the sum of the individual widths. This case is so simple that no demonstration is necessary.

Let us now make the width B constant throughout the length L, then for any length x we will have:

$$H^2 = \frac{6 P x}{B f}, \text{ whence } H = \sqrt{\frac{6 P x}{B f}} = K_1 x,$$

where $$K_1 = \text{constant} = \sqrt{\frac{6 P}{B f}}.$$

By means of this equation we can delineate the shape of the solid prism, the elevation of which is shown in

Fig. 27, the top and bottom contour lines being parabolas. Should one of the contour lines be made straight —as is often the case in practice—the other would be a parabola, but the hight of a section at a given point would remain the same in either case.

Since B is constant, if we divide the section into any number of rectangles, the sum of the widths of the

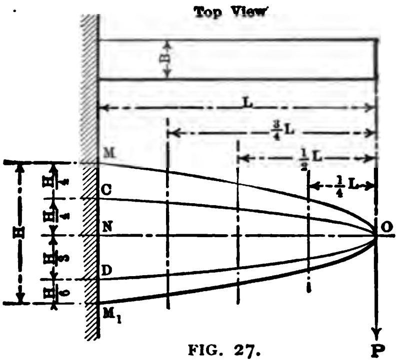


FIG. 27.

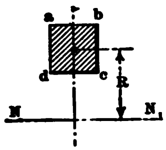


FIG. 28.

prisms thus formed will be a constant throughout the length and equal to B.

Before proceding further let us recall the equation expressing the moment of inertia of a section taken with respect to an axis lying in the same plane, as shown in Fig. 28. The center of gravity of section $a\,b\,c\,d$ is distant a length R from the given axis NN_1. In treatises on applied mechanics it is shown that the moment of inertia I taken with respect to NN_1 equals the inertia moment i of the section $a\,b\,c\,d$, plus the product of the

sectional area A by the square of the radius of giration R, or:

$$I = i + A R^2. \qquad (32)$$

As an example, let us find the moment of inertia of the rectangular section of the beam shown in Fig. 26. If we conceive the section as divided into two equal parts by the neutral axis $N N_1$, then, according to formula (32), we have for the upper part:

$$\frac{I}{2} = \frac{B}{12} \cdot \left(\frac{H^3}{2}\right) + B \cdot \frac{H}{2} \times \left[R^2 = \left(\frac{H}{4}\right)^2\right]$$

$$\frac{I}{2} = \frac{B H^3}{24}.$$

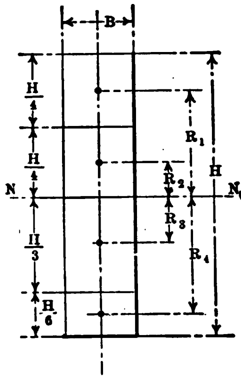


FIG. 29.

The moment of inertia of the lower part has the same value, and their sum is, as we know:

$$I = 2\left(\frac{I}{2}\right) = 2\left(\frac{B H^3}{24}\right) = \frac{B H^3}{12}.$$

If a rectangular section having a moment of inertia $I = \frac{B H^3}{12}$ was divided into four parts, as shown in Fig. 29, the moment of each part, beginning from the top, would be:

$$\frac{B H^3}{12 \times 64}, \quad \frac{B H^3}{12 \times 64}, \quad \frac{B H^3}{12 \times 27}, \quad \frac{B H^3}{12 \times 216}.$$

The radii of giration are respectively:

$$R_1 = \tfrac{3}{8} H,\ R_2 = \tfrac{1}{8} H,\ R_3 = \tfrac{1}{6} H,\ R_4 = \tfrac{5}{12} H.$$

The moments of inertia of these parts with respect to $N N_1$ are respectively:

$$i_1 = \frac{B H^3}{12 \times 64} + \frac{B H}{4} \times \frac{9}{64} H^2 = \frac{7}{16} \cdot \frac{B H^3}{12} = \frac{7}{16} I$$

$$i_2 = \frac{B H^3}{12 \times 64} + \frac{B H}{4} \times \frac{1}{64} H^2 = \frac{1}{16} \cdot \frac{B H^3}{12} = \frac{1}{16} I$$

$$i_3 = \frac{B H^3}{12 \times 27} + \frac{B H}{4} \times \frac{1}{36} H^2 = \frac{4}{27} \cdot \frac{B H^3}{12} = \frac{4}{27} I$$

$$i_4 = \frac{B H^3}{12 \times 216} + \frac{B H}{6} \times \frac{25}{144} H^2 = \frac{19}{54} \cdot \frac{B H^3}{12} = \frac{19}{54} I$$

$$\left.\begin{matrix}\text{Total} \\ I\end{matrix}\right\} = \ldots\ldots\ldots\ldots \frac{B H^3}{12} = I$$

We can easily conclude without further proof that the moment of inertia of a section, whatever may be its shape, is equal to the sum of the individual moments of its parts.

For the present case equation (1), $P x = f \frac{I}{a}$ may be written:

$$P x = \left(\frac{i_1 + i_2 + i_3 + i_4}{a}\right)$$

and if the section had been divided into a number n of

parts such that the moments of inertia of these with respect to NN_1 be equal to one another, we would have,

$$Px = f\frac{n\,i_n}{a}.$$

Hence, were it wanted to calculate the longitudinal dimensions of each prism, it would be correct to write:

$$\frac{P}{n}x = f\frac{i_n}{a}.$$

In other words, each part furnishes its quota of molecular resistance according to the value of its moment of inertia, and, corresponding to this, the bending moment for this part is equal to the total bending moment divided by the total moment of inertia and multiplied by the moment of inertia of the part considered. For instance, for the top part of the rectangular section just considered, we would have:

$$\frac{Px \times i_1}{I} = f\frac{i_1}{a} = \frac{7}{16}Px.$$

Taking again equation (1) for the maximum section we have:

$$PL = f\frac{I}{a}$$

and for any other section:

$$Px = f\frac{I_1}{a_1}.$$

whence

$$\frac{x}{L} = \frac{\left(\frac{I_1}{a_1}\right)}{\left(\frac{I}{a}\right)} \tag{33}$$

If we designate by i and i_1 the moments of inertia of a part corresponding respectively to L and x, we have:

$$\frac{i P L}{I} = f\,\frac{i}{a};\quad \frac{i P x}{I} = f\,\frac{i_1}{a_1}$$

whence by combining these expressions:

$$\frac{x}{L} = \frac{i_1}{i} \cdot \frac{a}{a_1} \tag{34}$$

comparing equations (33) and (34), we have:

$$\frac{x}{L} = \frac{I_1}{I} \cdot \frac{a}{a_1} = \frac{i_1}{i} \cdot \frac{a}{a_1}$$

whence

$$\frac{I_1}{I} = \frac{i_1}{i} \text{ or } \frac{i}{I} = \frac{i_1}{I_1}. \tag{35}$$

This point is exceedingly important, for we see now that for two positions L and x, the moments of inertia *total* I and I_1 and the partial moments i and i_1 bear the same ratio· thus, if we write $\frac{I_1}{I} = m$, we have also $\frac{i_1}{i} = m$.

If we call i_0 and i_0^1 the moments of inertia proper of the partial section referred to its own axis and corre-

sponding respectively to L and x, we have by equation (32):

$$i = i_0 + (\text{area} \times R^2),$$

$$i_1 = i_0^1 + (\text{area}_1 \times R^2_1).$$

Since

$$\frac{i_1}{i} = m, \quad i_1 = i\,m,$$

whence

$$i_1 = i_0 m + (\text{area} \times R^2)\,m.$$

Let us make

$$i_0 = \frac{b\,h^3}{12} \text{ and } i_0^1 = \frac{b_1\,h_1^3}{12} = i_0\,m.$$

Then

$$\frac{x}{L} = \frac{i_1\,a}{i\,a_1} = \frac{a}{a_1}\left(\frac{\dfrac{b_1\,h^3_1}{12} + A\,R^2\,m}{\dfrac{b\,h^3}{12} + A\,R^2}\right)$$

whence

$$a_1\,x\left(\frac{b\,h^3}{12} + A\,R^2\right) = a\,L\left(\frac{b_1\,h_1^3}{12} + A\,R^2\,m\right)$$

$$a_1\,x\left(\frac{b\,h^3}{12}\right) - a\,L\left(\frac{b_1\,h_1^3}{12}\right) = a\,L\,A\,R^2\,m$$

$$-\,A\,R^2\,a_1\,x = A\,R^2\,(a\,L\,m - a_1\,x);$$

but

$$a\,L\,m - a_1\,x = o,$$

hence

$$a_1\,x\left(\frac{b\,h^3}{12}\right) = a\,L\left(\frac{b_1\,h_1^3}{12}\right)$$

and, finally,

$$\frac{a_1 x}{a L} = \frac{b_1 h_1^2}{b h^2} = \frac{i_1}{i} = \frac{I_1}{I} = \frac{i_0^1}{i_0}.$$

Now, if we remark that:

$$I = \frac{B H^3}{12} \qquad i_0 = \frac{b h^3}{12} \qquad a = \frac{H}{2}$$

$$I_1 = \frac{B_1 H_1^3}{12} \qquad i_0^1 = \frac{b_1 h_1^3}{12} \qquad a_1 = \frac{H_1}{2}$$

and also that, in the present case, we have assumed the width B to remain constant, we arrive at the conclusion that:

$$\frac{x}{L} = \frac{H_1^2}{H^2} = \frac{h_1^2}{h^2}. \tag{36}$$

Generally it is preferable to use the expression:

$$\frac{x}{L} = \frac{B_1 H_1^2}{B H^2} = \frac{b_1 h_1^2}{b h^2} = \left(\frac{I_1}{a_1}\right)\left(\frac{a}{I}\right) \tag{37}$$

which means that *any partial section* b h, *however small, will vary throughout the length of the beam in the same proportion as the main section* B H. This curious conclusion would be of little value in the case of a beam of rectangular section, because the variation of the section—as a whole—at different points lengthwise can be easily plotted, but it becomes valuable when it can be proved to hold good for a beam of any transverse section, as will be illustrated in the second of the following exam-

ples. This ends the demonstration, but in order to help the understanding of it and also to show fully the great advantages to be derived from the application of this new law, two practical examples are given herewith.

FIRST EXAMPLE.

Let the main transverse section—at the point of support—of a beam of length L, *held horizontally at one end and holding at the other a load* P, *be rectangular and divided, as shown in Fig.* 29. *Find the dimensions of the partial and main sections at three places eorresponding respeetively to* $x = \frac{L}{4}, \frac{L}{2}, \frac{3L}{4}$, *the thickness* B *of the beam remaining constant.*

SOLUTION.

Fig. 27 represents a longitudinal elevation of the beam, lines MO and M_1O being the outlines of the beam; lines CO, DO and NO are the outlines of the partial longitudinal sections of the beam, corresponding to the partial-transverse sections. Line NO is also the neutral axis.

Let us find at first the hight H of the sections at the points considered:

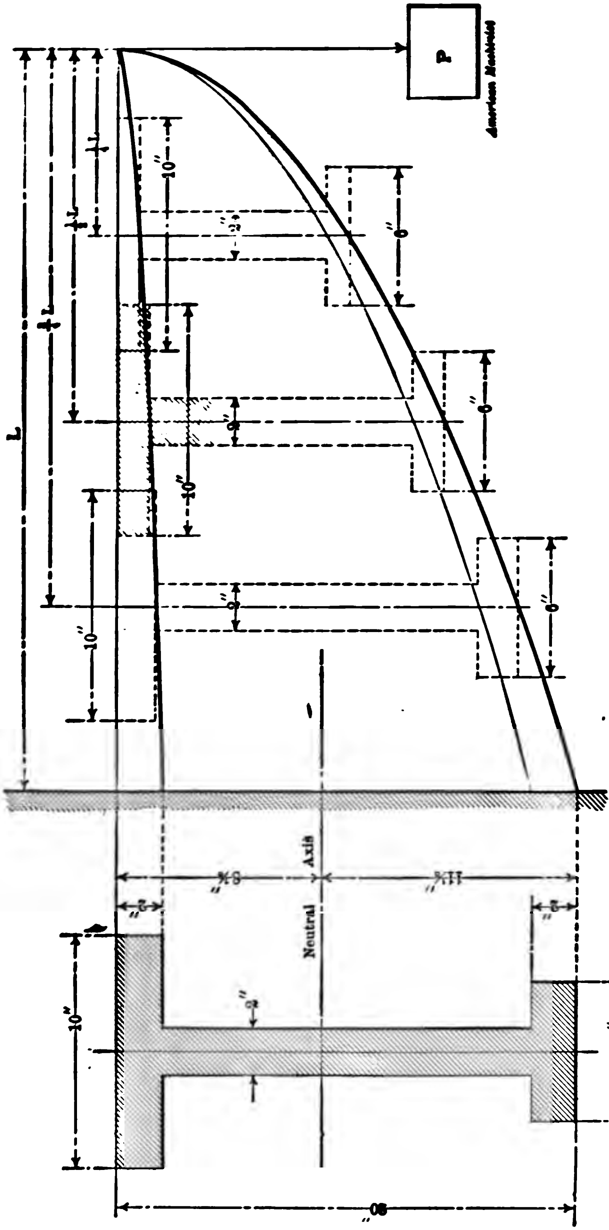


FIG. 31.

BEAMS OF UNIFORM STRENGTH.

FIG. 30.

The maximum bending moment is $PL = f\dfrac{BH}{6}$, and for any length x we have $Px = f\dfrac{B_1H_1^2}{6}$, whence

$$\frac{x}{L} = \frac{B_1H_1^2}{BH^2}.$$

The width B being constant, we have

$$\frac{x}{L} = \frac{H_1^2}{H^2}.$$

Solving for the unknown quantity H_1, we have:

$$H_1 = H\sqrt{\frac{x}{L}}.$$

Then, for $x =$	$\dfrac{L}{4}$	$\dfrac{L}{2}$	$\dfrac{3}{4}L$
$H_1 =$	$H\sqrt{\dfrac{1}{4}}$	$H\sqrt{\dfrac{1}{2}}$	$H\sqrt{\dfrac{3}{4}}$
$=$	$\dfrac{H}{2}$	$0.707\,H$	$0.866\,H$

Now, for any partial section, we also have

$$h_1 = H\sqrt{\frac{x}{L}}.$$

Hence for		$x =$	$\frac{L}{4}$	$\frac{L}{2}$	$\frac{3}{4}L$
Partial section	s_1	$h_1 =$	$\frac{H}{4} \times \frac{1}{2} = \frac{H}{8}$	$\frac{H}{4} \times 0.707 = 0.1768\,H$	$\frac{H}{4} \times 0.866 = 0.2165\,H$
	s_2	$h_2 =$	$\frac{H}{4} \times \frac{1}{2} = \frac{H}{8}$	$\frac{H}{4} \times 0.707 = 0.1768\,H$	$\frac{H}{4} \times 0.866 = 0.2165\,H$
	s_3	$h_3 =$	$\frac{H}{3} \times \frac{1}{2} = \frac{H}{6}$	$\frac{H}{3} \times 0.707 = 0.2357\,H$	$\frac{H}{3} \times 0.866 = 0.2886\,H$
	s_4	$h_4 =$	$\frac{H}{6} \times \frac{1}{2} = \frac{H}{12}$	$\frac{H}{6} \times 0.707 = 0.1278\,H$	$\frac{H}{6} \times 0.866 = 0.1443\,H$
Sum of partial hights . .		$H_1 =$	. . . . . $\frac{H}{2}$	. . . . . $0.707\,H$	. . . . . . $0.866\,H$

These figures are not absolutely necessary to establish the proof, but were needed for making the drawing of Fig. 27. The hight H being composed of four parts, if we multiply each of these by a common factor, the sum of the products is equal to the product of the whole hight by said factor; thus:

$$\left(\frac{H}{4} \times 2\right) + \left(\frac{H}{4} \times 2\right) + \left(\frac{H}{3} \times 2\right) + \left(\frac{H}{6} \times 2\right)$$

$$= \left(\frac{H}{4} + \frac{H}{4} + \frac{H}{3} + \frac{H}{6}\right) \times 2 = H \times 2.$$

SECOND EXAMPLE.

Let the main section of the beam be of I-*shape, as shown in Fig.* 30. *Find the dimensions of the main and partial sections for* $x = \frac{L}{4}, \frac{L}{2}, \frac{3}{4} L$. 1^0, *the width remaining constant;* 2^0, *the hight remaining constant;* 3^0, *the width varying in a determined manner.*

SOLUTION I.

A first glance at the proposed transverse section reveals the fact that the material comprising the beam resists in a different degree compressive or tensile

stresses. Effectively the material is so distributed throughout the section that the upper part—resisting tensile stresses—has a lesser maximum stress f_t than the lower part, which works in compression (f_c). (The intensity of the stresses developed in the beam varies as the distance of the part considered from the neutral axis.) But this fact does not hinder us in any way; on the contrary, it will render the proof of the law more palpable.

We may divide the main section into four parts : Two above the neutral axis, *i. e.*, the top flange (10 x 2 inches), the web (2 x $6\frac{7}{8}$)inches); and two below the axis, *i. e.*, the web (2 x $9\frac{1}{8}$ inches), and the lower flange (6 x 2 inches). Since it is required that the width is to remain constant, we will make the top and bottom flanges and the web respectively 10, 6 and 2 inches wide throughout the length.

We will multiply the main hight of each partial section successively by $\sqrt{\frac{3}{4}}$, $\sqrt{\frac{1}{2}}$, $\sqrt{\frac{1}{4}}$, and, with the products obtained as ordinates and $\frac{3}{4}L$, $\frac{1}{2}L$, $\frac{1}{4}L$ respectively as abscissæ, points will be determined through which contour lines will be drawn as shown in Fig. 31. The resulting transverse sections are shown in dotted lines at their respective places. The Tables of transverse sections given herewith prove beyond doubt that the simple method employed for this solution is correct.

It should be understood that the case just treated is

$x = L$					$x = \frac{3}{4}L$				
No.	B	H	R	I	No.	B	H	R	I
1	10"	2"	$7\frac{1}{8}$"	1246.976	1	10"	1.732"	6.82"	809.934
2	2"	$6\frac{1}{8}$"	$3\frac{1}{16}$"	216.634	2	2"	5.954"	2.977"	140.384
top total		$8\frac{1}{8}$"		1463.61			7.686"		950.318
3	2"	$9\frac{1}{8}$"	$4\frac{9}{16}$"	506.538	3	2"	7.902"	3.951"	329.003
4	6"	2"	$10\frac{1}{8}$"	1234.19	4	6"	1.732"	8.768"	801.63
bott. total		$11\frac{1}{8}$"		1740.723			9.634"		1130.633
$\frac{I}{a}$ required, top $= \frac{I}{a} \times \frac{x}{L} =$				164.91					123.69
$\frac{I}{a}$ calculated " $= \frac{I_1 + I_2}{H_1 + H_2} =$				164.91					123.69
$\frac{I}{a}$ required bott. $= \frac{I}{a} \times \frac{x}{L} =$				156.47					117.35
$\frac{I}{a}$ calculated " $= \frac{I_3 + I_4}{H_3 + H_4} =$				156.47					117.35

$x = \frac{1}{2}L$					$x = \frac{1}{4}L$				
No.	B	H	R	I	No.	B	H	R	I
1	10"	1.414"	5.508"	440.9	1	10"	1"	$3\frac{15}{16}$"	155.872
2	2"	4.861"	2.43"	70.592	2	2"	$3\frac{7}{16}$"	$1\frac{23}{32}$"	27.079
top total		6.275"		517.492			$4\frac{7}{16}$"		182.951
3	2"	6.452"	3.226"	179.086	3	2"	$4\frac{9}{16}$"	$2\frac{9}{32}$"	63.816
4	6"	1.414"	7.159"	436.352	4	6"	1"	$5\frac{1}{16}$"	154.28
bottom total 7.866				615.438			$5\frac{9}{16}$"		217.596
$\frac{I}{a}$ required, top $= \frac{I}{a} \times \frac{x}{L} =$				82.46					41.23
$\frac{I}{a}$ calculated " $= \frac{I_1 + I_2}{H_1 + H_2} =$				82.46					41.23
$\frac{I}{a}$ required bott. $= \frac{I}{a} \times \frac{x}{L} =$				78.23					39.12
$\frac{I}{a}$ calculated " $= \frac{I_3 + I_4}{H_3 + H_4} =$				78.23					39.12

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TABLES OF TRANSVERSE SECTIONS. EXAMPLE II, CASE I.

taken as an example only to prove the law, as the resulting shape may be found very inconvenient and difficult of execution.

SOLUTION 2.

The hight remaining constant, the widths of the flanges and web will taper gradually to a line at the place of application of the load, theoretically, just as in Fig. 26. The volume of the beam will then be a minimum, and by adopting this shape for the solution of the initial (simple) problem—see Part I.—our task is at an end.

SOLUTION 3.

The problem requires that the width vary in a determined manner. Let us assume that the widths of the flanges and web taper in the same and direct manner from B at the point of support to $\frac{B}{3} = B_0$ at the loaded end, or in the ratio of 3 to 1. For any length x the width B_1 will be:

$$B_1 = (B - B_0)\frac{x}{L} + B_0 \qquad (38)$$

The contour lines of the widths being straight, no calculations are necessary for delineating; but to obtain

the hights at various points equation (38) must be used—somewhat transformed—as will be seen presently.

From equation (37) we have:

$$\frac{B_1 H_1^2}{B H^2} = \frac{x}{L}$$

replacing B_1 by its value as above, and solving for H, we have:

$$H_1 = H \sqrt{\frac{B x}{(B - B_0)\, x + B_0 L}}. \qquad (39)$$

Thus the dimensions of the upper flange for $x = \frac{3}{4} L$, would be :

$$B_1 = (B - B_0)\,\frac{x}{L} + B_0$$

$$B_1 = \left(10 - \frac{10}{3}\right) \times \frac{3}{4} + \frac{10}{3} = 8\tfrac{1}{3}''$$

$$H_1 = 2 \sqrt{\frac{10 \times 0.75\, L}{\left(10 - \frac{10}{3}\right) \times \frac{3}{4} L + \frac{10}{3} L}} = 1.897''.$$

The other dimensions can be obtained just as easily as this one, and it is unnecessary to proceed further, for the method is too simple to need additional proof of its correctness.

If the widths were made to vary in a different manner, it would suffice to introduce in formula (37) the equation of the variation of the width—whatever it might be—and to solve for H_1, as we have done.

Graphical processes may be used with advantage instead of calculations, and much time can be saved thereby.

We can easily see now that all the shapes convenient for beams of uniform strength and of rectangular section are also suitable for beams having complex sections, *provided all the parts of the sections be treated in the same manner throughout the beam.*

In concluding this long series of articles the writer believes that the solution of the *simple problem* is as complete in its elementary form as up-to-date knowledge can make it. But, after all, the problem was only used as a pretext to show the necessity of making conscientious investigations in order to increase our knowledge of the laws governing the resistance of materials.

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